JOURNAL OF FINANCIAL INTERMEDIATION 2, 168–206 (1992)

Anonymity in Securities Markets*

MARGARET M. FORSTER AND THOMAS J. GEORGE

College of Business, The Ohio State University, Columbus, Ohio 43210

Received December 3, 1991

We analyze how the anonymous trading of uninformed agents affects the characterization of security market equilibrium. We show that the degree of anonymity provided by a market alters the distribution of wealth across agents, the depth of the market, and the incentive agents have to acquire private information about a security's fundamental value. Moreover, the nature of these effects depends on the type of information about uninformed trading that is revealed to market participants. Our results have implications for sunshine trading, dual trading, brokerage relationships, automation and decentralization of markets, and firms' security listing choices. *Journal of Economic Literature* Classification Numbers: D82, G10, L10. © 1992 Academic Press, Inc.

1. INTRODUCTION

In most of the literature relating to information and the structure of securities markets, a subset of the agents in the market is privately informed about the fundamental value of the traded security while the rest are uninformed. Informed agents' optimal trading choices depend on their beliefs concerning the net orders to be executed by uninformed agents who trade for an exogenous liquidity need [see, for example, Glosten and Milgrom (1985), Kyle (1985), Easley and O'Hara (1987), Admati and Pfleiderer (1988, 1991), Glosten (1989), Foster and Viswanathan (1990, 1991), Lindsey (1990), Madhavan (1990), Pichler (1990), Roell (1990), Fishman and Hagerty (1991), Kumar and Seppi (1991), and Subrah-

* We thank Mark Bagnoli, James Dow, Michael Fishman, Carol Frost, Chuan-Yang Hwang, Ananth Madhavan, George Oldfield, Anthony Sanders, René Stulz, and an anonymous referee for especially helpful suggestions. Helpful comments were also received from seminar participants at Carnegie-Mellon University, The Ohio State University, and the 1991 European Finance Association Meetings. Errors are our responsibility.

1042-9573/92 $5.00

manyam (1991); for a survey see Black (1992)]. In this paper, we study how the characterization of security market equilibrium is affected when market participants learn something in advance about the nature of the uninformed trades to be executed. If something about the nature of these trades is revealed prior to order submission, then trading must not be fully anonymous. We consider how anonymity about uninformed trades influences the distribution of wealth across agents, market liquidity, the informativeness of prices, and the endogenous acquisition of private information about the fundamental value of a traded security.

We highlight the need to analyze the effect of anonymity on securities market equilibrium by noting that several institutional arrangements exist whereby market participants learn something in advance about the nature of agents' orders. Sunshine trading, for example, involves direct disclosure by an agent of information regarding his future trades. These disclosures can change market participants' beliefs regarding the information content of order flow. In centralized markets, agents can observe the other agents present on the trading floor. If identities confer information about motivations for trade, then a centralized market allows agents to have advance knowledge of the nature of orders to be executed. Brokerage relationships enable brokers to learn their customers' motivations for trading. Brokers can use this information themselves by dual trading or pass this information on to other customers prior to order submission as happens, for example, by "shopping" block trades. A further example is provided by access to the book of public limit orders. The contents of the book convey information about the orders that are to be executed at various prices. Whether this information is public or closely held affects the extent to which market participants have advance knowledge of the nature of trades to be executed. In addition, our results are relevant for market designers attempting to attract trading volume and firms' securities listings, because the anonymity feature of a market's design affects the distribution of wealth across agents, market depth, and the incentives for acquiring private information about a security's fundamental value.

Since agents who have private information about the security's fundamental value have the greatest incentive to conceal their identities, we focus on the anonymous execution of liquidity-motivated trades. Anonymous order execution implies that neither the direction nor the magnitude of liquidity trades is known prior to trading. We analyze the effects of revealing the direction and magnitude separately. This enables us to specifically address what information would be revealed by a sunshine trader or used by a dual trader and what information should be disclosed by an automated trading system or be publicly revealed from the book of limit orders. Moreover, the structure of the economy is held fixed across our comparisons in order to isolate the effects due only to changing the allocation of information about liquidity trading.

It is reasonable to expect that anonymity is a matter of degree and that knowledge of traders' motivations need not be widely disbursed. To account for this as completely as possible, we consider cases in which knowledge of liquidity trade is revealed: (i) to all market participants, (ii) only to those making the market, (iii) only to those who are trading for profit, and (iv) the fully anonymous case in which the nature of liquidity trading is revealed to no one. The first and last cases enable us to make statements about market designs in which anonymity is entirely absent or fully present; the intermediate cases enable us to recognize that certain features of a market's design can give some agents informational advantages because of their position in the market or because of their relationships with customers whose trades they execute. Across these four cases, we compare several ex-ante attributes of equilibrium: the distribution of expected wealth across agents, the sensitivity of prices to order flow, price informativeness, and the incentives provided for the acquisition of private information about the security's fundamental value.

We show that revealing the direction of liquidity trade (i.e., net buy or net sale) in advance of trading: (i) can decrease the expected transactions costs borne by liquidity-motivated traders; (ii) does not affect the average sensitivity of prices to order flow or price informativeness; and (iii) reduces the incentive for private information acquisition, provided that there is at least one privately informed agent. These results suggest, for example, that revealing net order imbalances to all market participants prior to trading, as happens in trading halts, can decrease the cost of transacting for liquidity traders. In addition, the practice of dual trading has a negative impact on agents' incentives to acquire private information about a security's fundamental value.

We show that revealing the absolute magnitude of liquidity trade in advance of trading: (i) decreases the transaction costs borne by liquidity traders if, and only if, there is sufficient competition among agents who are privately informed; (ii) increases the average sensitivity of prices to order flow; (iii) does not affect average price informativeness; and (iv) increases the incentive for acquiring costly private information that is possessed by few traders and discourages the acquisition of inexpensive information that is widely possessed. These findings suggest, for example, that an open limit order book which allows all traders to estimate the volume of trading at different prices, encourages the production of new information.

Although related, our analysis differs from models that study the behavior of discretionary liquidity traders [for example, Admati and Pfleiderer (1988, 1989) and Foster and Viswanathan (1990)]. These analyses focus on comparing periods of concentrated trading to periods without concentrated trading. In equilibrium, trading concentrates in certain time periods because the timing of agents' orders is common knowledge. By contrast, we examine how equilibria in which the nature of agents' orders is commonly known differ from equilibria in which the nature of these orders is not known. Our comparisons enable us to analyze the implications of a policy, or feature of market design, that reveals information about liquidity trade.

Admati and Pfleiderer (1991) and Roell (1990) present models in which a subset of the agents observe a noisy signal of the liquidity trade component of order flow. Although these papers differ in their assumptions concerning risk aversion and the competitiveness of traders, both conclude that transmission of such a signal enhances the welfare of the liquidity traders. This is because a noisy signal decreases the posterior variance of agents' beliefs about liquidity trades. In both models, this increases the sensitivity of prices to order flow by enough to cause a wealth transfer from informed agents to the uninformed agents. While this is an important class of information revelations, restricting attention to it limits our ability to examine disclosures that cause agents to revise upward their beliefs concerning the variance of liquidity trade (as might happen, for example, if participants in a centralized market observe that an abnormally large number of traders are present on the trading floor). To account for this in a simple way, we model liquidity trade as a mixture of distributions. This makes it possible for agents to revise upward their beliefs concerning the variance of liquidity trade. If information is revealed about liquidity order flow prior to trading, the revelation tells market participants from which distribution liquidity trade will be drawn.

Madhavan (1990) compares the equilibria of two specific trading mechanisms that differ in the extent of anonymity they provide. In the first, agents' order placement strategies are common knowledge; in the second, agents' orders are common knowledge. Differences in the characterization of equilibrium across markets arise because the market structures differ in the extent to which agents can estimate the private information of other agents. Our analysis holds knowledge of informed agents' orders fixed across markets and focuses instead on differences associated with changing the allocation of information concerning only liquidity trades.

Fishman and Hagerty (1991) and Kumar and Seppi (1991) present multiperiod models that can be interpreted as analyses of the anonymity issue in their respective contexts. In a futures market with cash settlement, Kumar and Seppi show that if informed trade varies across periods in a commonly known way, then uninformed agents can manipulate prices to make profits. Fishman and Hagerty show that if the reporting of insiders' trades is mandatory then in periods when insiders are less informed than expected, their disclosures can be used to manipulate prices. Unlike our model, their models relax anonymity regarding trades made by informed or potentially informed agents and examine the possibility for manipulation implied by the intertemporal structure of the market.

The plan of the paper is as follows. Section 2 describes the market environment and the way we model anonymity. Section 3 examines equilibria where anonymity concerns the direction of liquidity trade, and Section 4 examines equilibria where anonymity concerns the magnitude of liquidity trade. Section 5 concludes. Proofs of all propositions and explicit expressions for parameter values are in the Appendix.

2. THE MODEL

We model the market for a single risky security whose random terminal payoff is denoted $\tilde{v}$. To isolate the direct effects of anonymity, we assume that all agents are risk neutral. A subset A of these agents includes those who are privately informed about the security's payoff because they observe the realization of information about the security's value, $\tilde{y} = \tilde{v} + \tilde{\varepsilon}$. A distinct subset B consists of agents who have no private information about the security's value. There is also a group of agents who have an exogenous liquidity need that motivates them to trade a net amount of x units of the security. Except for these liquidity-motivated agents, all other agents' motivation for trade is to maximize expected profits conditional on their private information about the security's value and whatever knowledge they might have about liquidity trading. We refer to these agents, the union of sets A and B, as profit maximizing agents. For concreteness, let there be M profit maximizing agents, and let $N(< M)$ of them be privately informed about the value of the security. For tractability, we assume that the random variables $\tilde{v}$ and $\tilde{\varepsilon}$ are mutually independent and normally distributed, with unconditional means of zero and unconditional variances of σ_v and σ_ε.

While potentially interesting, modeling the specific trading mechanisms that reveal the direction or the magnitude of liquidity trade is beyond the scope of our investigation. Our goal is to analyze the effects of such revelations on the characterization of equilibrium. Therefore, we simply assume that the trading mechanism is able to transmit a signal to market participants regarding either the direction or magnitude of the net amount of liquidity trade. This signal need not impart perfect information. All that is needed is that it causes a revision in agents' beliefs regarding the direction or magnitude of liquidity trade to more closely reflect the true state of liquidity trade. In a fully anonymous market, no such revision in beliefs occurs.

We model revelation of the *direction*, $E[\tilde{x}]$, of liquidity trade by assuming that the unconditional distribution of the net liquidity order, $\tilde{x}$, is a mixture of normal distributions

$$\tilde{x} \sim \begin{cases} N(\mu_H, \sigma + \mu_H^2) & \text{with probability } \frac{1}{2} \\ N(\mu_L, \sigma + \mu_L^2) & \text{with probability } \frac{1}{2}. \end{cases}$$

In a market that is less-than-fully anonymous regarding the direction of liquidity trade, the value of $\mu_x \in \{\mu_H, \mu_L\}$ is revealed to a subset of agents before trading. We assume that $\mu_H = -\mu_L$. This is not without loss of generality, but it allows us to focus on revealing the direction of liquidity trade while holding its magnitude fixed. This is because $\mu_H = -\mu_L$ implies that the conditional variance of $\tilde{x}$ does not depend on the realized value of μ_x.

We model revelation of the *magnitude*, $E[\tilde{x}^2]$, of liquidity trade by assuming that the unconditional distribution of the net liquidity order is a mixture of normal distributions

$$\tilde{x} \sim \begin{cases} N(0, \sigma_H) & \text{with probability } \frac{1}{2} \\ N(0, \sigma_L) & \text{with probability } \frac{1}{2}, \end{cases}$$

where $\sigma_H > \sigma_L > 0$. In a market that is less-than-fully anonymous regarding the magnitude of liquidity trade, the value of $\sigma_x \in \{\sigma_L, \sigma_H\}$ is revealed to a subset of agents before trading. To isolate the effect of a revelation of magnitude, we assume that the conditional expectation of $\tilde{x}$ is invariant to the realization of σ_x (i.e., it equals zero for either realization). The parameter $\bar{\sigma}$ denotes the unconditional variance, $\bar{\sigma} = \frac{1}{2}\sigma_H + \frac{1}{2}\sigma_L$. The benefit obtained by using mixtures of normal distributions is that we are able to analyze direction and magnitude separately, and the results differ.

We assume that the random variables μ_x and σ_x are independent of $\tilde{v}$ and $\tilde{\varepsilon}$ and that conditional on μ_x or σ_x, $\tilde{x}$ is independent of $\tilde{v}$ and ε. For revelations of either direction or magnitude, we define q to be the probability that the M profit-maximizing agents' beliefs are revised to the conditional mean or conditional variance of liquidity trade. In solving for equilibrium, q is left unrestricted, but our comparisons across markets focus on the cases where $q = 0$ or $q = 1$. We let γ_x be the generic notation for the parameter of the distribution of $\tilde{x}$ about which market participants might have advance knowledge; thus, $\gamma_x \in \{\mu_x, \sigma_x\}$.

2a. *Market Equilibrium*

Conditional on private information about security value and knowledge of liquidity trade, trading of the risky security is modeled similar to Kyle (1985). Each profit-maximizing agent, i, chooses an order, $\psi_i = \Psi_i(\Phi_i; P)$, as a function of his information regarding security value and liquidity trade, Φ_i, and as a function of the pricing rule employed by the marketmaker, P. The marketmaker observes order flow $\omega \equiv \sum_{i=1}^{M} \psi_i + x$ and sets a price $p = P(\omega; \Psi, \Phi_o)$ that is based on his expected value of the security conditional on order flow, the trading rules employed by the agents, $\Psi = \{\Psi_1, \ldots, \Psi_M\}$, and the marketmakers' knowledge of liquidity trade, Φ_o. An equilibrium in the securities market is defined as follows.

DEFINITION 1. A securities market equilibrium is a vector $(\psi_1, \ldots, \psi_M, p)$ such that agents' orders maximize expected profits

$$E[\pi_i(\psi_i)|\Phi_i; N, q] \geq E[\pi_i(\hat{\psi}_i) \mid \Phi_i; N, q] \qquad \text{for all } \hat{\psi}_i,\ i = 1, \ldots, N,$$

where $\pi_i(\cdot)$ denotes agent i's profit. If γ_x is revealed to the marketmaker, the price is set equal to the expected value of $\tilde{v}$ conditional on order flow; if γ_x is not revealed to the marketmaker the price is set equal to the average of these conditional expectations

$$p = \begin{cases} E[\tilde{v}|\omega; \Psi, \gamma_x] & \text{if } \gamma_x \in \Phi_o \\ \frac{1}{2}E[\tilde{v}|\omega; \Psi, \gamma_H] + \frac{1}{2}E[\tilde{v}|\omega; \Psi, \gamma_L] & \text{if } \gamma_x \notin \Phi_o. \end{cases}$$

If γ_x is revealed to the marketmaker, our definition of equilibrium is identical to that of Kyle (1985). Otherwise, we adopt a different definition of the equilibrium price. To maintain tractability, we assume that the marketmaker does not update his beliefs concerning γ_x from order flow—i.e., he retains his prior beliefs concerning γ_x in setting the price. This assumption yields a linear pricing rule and preserves the feature of behavior that is essential to this case—when γ_x is not revealed to the marketmaker, the price he sets reflects less than perfect information about γ_x.[1] Our most interesting results assume that γ_x is revealed to the marketmaker and are therefore unaffected by this assumption. The main reason for including cases where γ_x is not revealed to the marketmaker is to enable us to discuss dual trading for which the marketmaker is at an informational disadvantage relative to agents concerning the state of liquidity trade. Our assumption has the effect of making this informational disadvantage severe by not allowing the marketmaker to update his beliefs about liquidity trade based on his observation of order flow.

2b. *Measures of Market Quality*

Throughout the paper, we restrict our attention to pricing strategies that are linear functions of order flow and trading strategies that are linear functions of traders' private information.[2] Within this class of equilibria there is a unique equilibrium for each market we analyze, and this equilibrium is symmetric—$\Psi_i(\cdot) = \Psi_j(\cdot)$ for all i and j in set A, and $\Psi_k(\cdot) = \Psi_l(\cdot)$

[1] Pichler (1990) also makes this assumption. In a different context, Hughson (1988) presents numerical results that indicate that nonlinearities in pricing functions created by setting the price equal to the marketmaker's conditional expected value may not be significant. We are grateful to the referee for pointing this out.

[2] Definition 1 makes this possible even when the marketmaker does not observe the direction or magnitude of liquidity trade. However, this restriction rules out nonlinear (option-like) trading strategies.

for all k and l in set B. Consequently, the pricing rule of the marketmaker and the trading strategies of agents take the form

$$P(\omega; \Psi) = \theta_{\gamma_x} + \lambda_{\gamma_x}\omega, \qquad \Psi(\Phi_i^A; P) = \beta_{\gamma_x} + \alpha_{\gamma_x} y, \qquad \Psi(\Phi_i^B; P) = \delta_{\gamma_x},$$

where the subscripts refer to the realized value of γ_x; a subscript of zero denotes a parameter value obtained when γ_x is not observed by the agent to whom that parameter relates. The λ parameter measures the sensitivity of the equilibrium price to additional units of order flow. Therefore, it can be used as an inverse measure of market depth or liquidity [see, for example, Kyle (1985), Admati and Pfleiderer (1988), Foster and Viswanathan (1990), and Subrahmanyam (1991)]. Part of our analysis will examine the impact that the anonymity feature of a market's design has on this parameter.

Our model also enables us to address the extent to which anonymity affects the distribution of profits across market participants. For this purpose, we compare the expected profits of privately informed and uninformed liquidity traders across equilibria. Since uninformed traders are necessary for a deep or liquid market, the effect that anonymity has on the allocation of wealth is important to policymakers.

Another issue we analyze is the extent to which the equilibrium security price reflects the security's fundamental value. This is important because it indicates the quality of a market as a price discovery mechanism. Since we compare markets that differ in the anonymity they provide, this statistic enables us to assess the impact of anonymity on price discovery. To measure this, we define price informativeness as follows.

DEFINITION 2. For a given q, price informativeness is the extent to which an observation of the equilibrium price decreases an uninformed agent's uncertainty about the security's payoff, $\tilde{v}$:

$$\Delta(q) = \mathrm{Var}[\tilde{v}] - \mathrm{Var}[\tilde{v} \mid P; q].$$

Of particular importance in this paper is the complementary nature of information about liquidity trade and information about the fundamental value of the security. Therefore, we analyze the incentive provided by revealing the direction or magnitude of liquidity trade for the acquisition of private information regarding security value. If the cost of becoming informed is $c \geq 0$, we define an equilibrium in information acquisition as follows.

DEFINITION 3. For a given q, an equilibrium in private information acquisition is a nonnegative integer $N^*(q; c)$ such that if $N^*(q; c)$ agents acquire information about security value, each uninformed agent prefers to remain uninformed:

$$N^*(q;\, c) = \left\{ \begin{array}{l} N \in \mathrm{N} : E[\pi(\psi_i^A)|\Phi_i^A;\, N,\, q] - c \geq E[\pi(\psi_i^B)|\Phi_i^B;\, N-1,\, q] \\ \quad \text{and } E[\pi(\psi_i^A)|\Phi_i^A;\, N+1,\, q] - c < E[\pi(\psi_i^B)|\Phi_i^B;\, N,\, q] \end{array} \right\}.$$

The next two sections analyze the effects of anonymity on the characterization of equilibrium.

3. ANONYMITY REGARDING THE DIRECTION OF LIQUIDITY TRADE

In this section, we model less than full anonymity by providing market participants with some knowledge of the direction of liquidity trade; we assume that their expectation of net liquidity trade is revised from zero to $\mu_x \in \{\mu_H, \mu_L\}$. The assumption that μ_H and μ_L are equally likely implies that agents' beliefs about the magnitude of liquidity trade remain unchanged. First we analyze the case in which the marketmaker's expectation is revised to μ_x and then the case in which it is not. In both cases, the expectations of the M profit-maximizing agents are revised with probability q. Our analysis of anonymity compares behavior in the equilibria in which q takes the values of zero and one for each case—i.e., in equilibria where there is no uncertainty about whether agents' beliefs are revised. This is followed by a comparison of the two cases relating to whether or not the direction is revealed to the marketmaker. There we discuss the informational aspects of these equilibria.

3.1. *Marketmaker Observes* μ_x

We assume that whether μ_x takes the value μ_H or μ_L is revealed to the marketmaker.[3] There are N agents who observe y and $(M - N)$ agents who do not observe y. All M profit-maximizing agents observe μ_x with probability q. In this context, there exists an equilibrium with linear trading strategies and a linear pricing rule.[4]

PROPOSITION 3.1. *There exists an equilibrium in which*

$$P(\omega;\, \Psi) = \theta_{\mu_x} + \lambda_{\mu_x}\omega, \qquad \Psi(\Phi_i^A;\, P) = \beta_{\mu_x} + \alpha_{\mu_x} y, \qquad \Psi(\Phi_i^B;\, P) = \delta_{\mu_x}.$$

[3] Madhavan and Smidt (1992) present empirical evidence indicating that NYSE specialists have advance knowledge of future order imbalances and that this affects their pricing behavior.

[4] All security market equilibria we analyze are unique in the class of linear equilibria. Verifying uniqueness in some cases involves tedious calculations that have not been included in the Appendix, but are available from the authors.

Agents' optimal expected profits are

$$E[\pi_i(\psi_i^A)|\Phi_i^A; N, q] = \frac{I_v \sigma_v}{\lambda(N+1)^2}$$

$$E[\pi_i(\psi_i^B)|\Phi_i^B; N, q] = 0,$$

where $I_v = \sigma_v/(\sigma_v + \sigma_\varepsilon)$ *and* $\lambda = \lambda_{\mu_H} = \lambda_{\mu_L}$.

In order for the profit-maximizing agents to benefit from knowing the direction of liquidity trade, the marketmaker would have to confuse this liquidity trade for an order imbalance caused by informed trading. However, since the value of μ_x is revealed to the marketmaker, he adjusts the price to fully reflect this by setting $\theta_{\mu_x} = -\lambda\mu_x$. Consequently, even if liquidity trade is revealed to the profit-maximizing agents, they cannot profit by trading on their information about liquidity trade at these prices. As a result, the $(M - N)$ agents who are not privately informed about the security's fundamental value submit zero orders and earn zero expected profits in equilibrium, and the N privately informed agents' orders are based entirely on their private information ($\delta_{\mu_x} = \beta_{\mu_x} = 0$). In addition, these orders are the same whether or not liquidity trade is revealed to the privately informed. This is because agents' beliefs about the magnitude of liquidity trade do not depend on its direction (by construction). Likewise, the profit derived from trading on private information about the security's fundamental value is the same regardless of whether the direction of liquidity trade is revealed to the privately informed agents.

3.2. *Marketmaker Does Not Observe* μ_x

We now assume that the marketmaker does not know whether μ_x takes the value μ_H or μ_L. The N agents who observe y and the $(M - N)$ agents who do not observe y, observe μ_x with probability q. In this context, there exists an equilibrium with linear trading strategies and a linear pricing rule.

PROPOSITION 3.2. *There exists an equilibrium in which*

$$P(\omega; \Psi) = \theta_o + \lambda_o\omega, \qquad \Psi(\Phi_i^A; P) = \beta_{\mu_x} + \alpha_{\mu_x} y, \qquad \Psi(\Phi_i^B; P) = \delta_{\mu_x}.$$

Agents' optimal expected profits are

$$E[\pi_i(\psi_i^A)|\Phi_i^A; N, q] = E[\pi_i(\psi_i^B)|\Phi_i^B; N, q] + \frac{I_v \sigma_v}{\lambda_o(N+1)^2}$$

$E[\pi_i(\psi_i^B)|\Phi_i^B; N, q]$

$$= \frac{1}{(M+1)^2}\left\{q\lambda_o\left[\frac{1}{2}\left(\frac{\theta_o}{\lambda_o} + \mu_H\right)^2 + \frac{1}{2}\left(\frac{\theta_o}{\lambda_o} + \mu_L\right)^2\right] + (1-q)\frac{\theta_o^2}{\lambda_o}\right\},$$

where $I_v \equiv \sigma_v/(\sigma_v + \sigma_\varepsilon)$.

Provided that there is an agent who has superior information about security value (i.e., $N \geq 1$), the marketmaker will confuse liquidity trade for an order imbalance caused by informed trading. If agents know more about the direction of liquidity trade than the marketmaker ($q = 1$), they can profit from the temporary price movement arising from the marketmaker's confusion. This is possible regardless of whether the agent has private information about the value of the security ($\delta_{\mu_x} = \beta_{\mu_x}$).

Since the direction of liquidity trade is not revealed to the marketmaker, the sensitivity of prices to order flow in equilibrium cannot depend on this direction. This sensitivity determines the profitability of trading on information about fundamentals. Therefore, the component of informed agents' orders relating to their private information is unaffected by whether the direction of liquidity trade is revealed to them ($q = 1$) or not ($q = 0$).[5] This, in turn, implies that equilibrium expected profits of privately informed agents have two components. The first derives from knowledge of liquidity trade and is equal to the equilibrium expected profits of the ($M - N$) agents who are not privately informed. The second derives from having private information about the security's fundamental value and is independent of whether liquidity trade is revealed or not.

3.3. *Informational Aspects of Equilibrium*

In this subsection we compare the two cases presented above. Differences between these cases highlight the relevance of the anonymity feature of a market's design. A two-part observation follows immediately from the expressions for agents' equilibrium expected profits. First, the private information component of equilibrium expected profits is identical for the two cases presented above regardless of whether $q = 0$ or $q = 1$. This means that the profitability of private information about fundamentals is unaffected by differences in anonymity provided by markets con-

[5] The price set by the marketmaker does not depend on μ_x since he does not observe it. However, prices do depend on q, for $q \in (0, 1)$, through the parameter λ. This is because when $q \notin \{0, 1\}$, the marketmaker is uncertain about whether agents' beliefs about the direction of liquidity trade have changed to μ_x or remained at zero. If $q \in \{0, 1\}$, no such uncertainty exists and the expression for λ is simplified. The cases $q \in \{0, 1\}$ correspond to full and less-than-full anonymity. While it may be interesting to consider market designs that *randomly* provide different levels of anonymity, we focus here on $q \in \{0, 1\}$.

cerning the direction of liquidity trade. Second, if the direction of liquidity trade is not revealed to the marketmaker but is revealed to profit-maximizing agents, the expected profits of these agents contain a component that relates to their knowledge of the direction of liquidity trade. In addition, this component of expected profits is independent of whether the agent has private information about the security's fundamental value. This implies that anonymity concerning the direction of liquidity trade can affect the distribution of wealth across agents. Proposition 3.1 and 3.2 indicate that if the direction of liquidity trade is not revealed to the marketmaker then profit maximizing agents' expected profits are greater when liquidity trade is revealed to them ($q = 1$) than when it is not ($q = 0$).

The expected losses of the liquidity-motivated agents equal the expected profits of the profit-maximizing agents and the marketmaker.[6] These losses are greatest when neither profit-maximizing agents nor the marketmaker observe μ_x, less when profit-maximizing agents observe μ_x but the marketmaker does not, and smallest when the marketmaker observes μ_x.[7] This implies that, if given the choice, liquidity-motivated agents prefer to trade in an environment that reveals the direction of their orders to all market participants. If the direction of their orders is not revealed to the marketmaker, liquidity agents lose because the price does not reflect the direction of their trades. By revealing the direction of liquidity trades to the marketmaker or to other agents, the price at which liquidity-motivated orders are executed is more favorable given the direction of their net trade. This implies, for example, that disclosing the sign of the order imbalance at different prices contained in the limit order book can decrease the transaction costs faced by the liquidity-motivated agents.

Evaluating the expressions for the λ parameters in Propositions 3.1 and 3.2 at $q = 0$ and $q = 1$ indicates that these parameters are independent of the revelation of liquidity trade (i.e., $\lambda_o = \lambda_{\mu_H} = \lambda_{\mu_L}$ for $q = 0$ or $q = 1$). The λ parameter determines the sensitivity of prices to a unit of order flow in equilibrium, and this sensitivity is an inverse measure of market depth. Therefore, these results imply that the anonymity provided by a market for the direction of liquidity trade does not affect the depth of the market.

[6] When μ_x is not revealed to the marketmaker he is assumed not to update his beliefs concerning the distribution of liquidity trade based on his observation of order flow (Definition 1). Since the marketmaker does not know from which distribution liquidity trade is drawn, the price he sets is biased against liquidity traders. This leads liquidity traders to either "sell low" or "buy high," which yields positive unconditional expected profits for the marketmaker. A tedious calculation shows that $\lambda_o[1 - q(M/(M + 1))]^2 \mu_h^2$ is the marketmaker's unconditional expected profit for $q \in \{0, 1\}$. Later we discuss the implications of this for our results.

[7] This is easily verified using the previous footnote and the expressions for agents' profits given in Propositions 3.1 and 3.2.

Price informativeness reflects the precision with which the security payoff, $\tilde{v}$, can be inferred from the security price. Equality of λ parameters and equality of the private-information components of agents' orders across cases imply our next result.

PROPOSITION 3.3. *Price informativeness is independent of whether, and to whom, the direction of liquidity trade is revealed.*

Our analysis so far has been based on the assumption that the number of privately informed agents is exogenous. However, revealing the direction of liquidity trade affects agents' incentives to acquire information about fundamental value. If the direction of liquidity trade is not revealed to the marketmaker, agents can earn profits from knowing this direction. This knowledge enables them to forecast the temporary component in prices that exists because the marketmaker cannot distinguish between informed and liquidity trade when he observes net order flow. The number of agents who are privately informed about fundamentals is itself a determinant of the size, and hence profitability, of this temporary component in prices.

In a fully anonymous market, profit-maximizing agents do not observe the direction of liquidity trade and make their information acquisition decision based only upon the profitability of trading on information about fundamentals. Alternatively, if the direction of liquidity trade is revealed to them, these agents make their information acquisition decision based on the profitability of trading on information about fundamentals *and* the effect that such trading has on the profitability of their knowledge of the direction of liquidity trade.

PROPOSITION 3.4. (a) *Suppose the direction of liquidity trade is revealed to the marketmaker, then the equilibrium in information acquisition is independent of whether $q = 0$ or $q = 1$, i.e., $N^*(1; c) = N^*(0; c)$.*

(b) *Suppose the direction of liquidity trade is not revealed to the marketmaker and the equilibrium in information acquisition is unique.*

(i) *If $\lambda_o(N^*(1; c)) \geq \lambda_o(N^*(1; c) - 1)$ then $N^*(0; c) \leq N^*(1; c)$, alternatively*

(ii) *if $\lambda_o(N^*(1; c)) \geq \lambda_o(N^*(1; c) + 1)$ then $N^*(0; c) \geq N^*(1; c)$,*

(iii) *otherwise $N^*(0; c) = N^*(1; c)$.*

To interpret part (a), note that when the direction of liquidity trade is revealed to the marketmaker, whether agents know this direction or not has no effect on their equilibrium expected profits. Consequently, it does not affect their incentive to acquire private information. The interpretation of part (b) is that if λ_o is increasing in N, then revealing the direction of liquidity trade tends to increase the number of agents who are privately informed in equilibrium (relative to the number that would be informed in a fully anonymous market). This is because if λ_o is increasing in N, the temporary component in prices is larger the more privately informed agents there are. Alternatively, if λ_o is decreasing in N, the temporary component in prices is smaller the more privately informed agents there are, and less agents become privately informed when the direction of liquidity trades is revealed. This is because, at the margin, becoming informed decreases *total* profits. It is possible to identify which alternative prevails. Provided there is some informational asymmetry concerning the security's fundamental value, λ_o is *decreasing* in N [see Lemma 3.5 in the Appendix]. Therefore, revealing the direction of liquidity trade tends to *decrease* agents' incentives to acquire private information about security value.

Taken together, these results imply that revealing the direction of liquidity trade, but not its magnitude, has a limited effect on the informational aspects of equilibrium when N is fixed. Revealing this direction does not affect the depth of the market or the informativeness of equilibrium security prices. However, if N is endogenous and the marketmaker is not aware of the direction of liquidity trade, revealing this direction to agents decreases their incentive to acquire private information about the security's fundamental value. This is because agents recognize that becoming informed reduces the profitability of their information about liquidity trade.

Our results imply that a sunshine trader's revelation to the entire market of the direction of the impending trade decreases his cost of transacting. This revelation eliminates losses because the marketmaker sets a price that reflects the direction of the impending trade. Consequently, these trades are not executed at prices that are biased estimates of value conditional on the direction of liquidity trade.[8]

Further implications of these results concern the practice of dual trading—brokers trading for their own account on the basis of information inferred from customers' orders. A broker can trade based on information inferred from the trades of his privately informed customers, compete with those customers, and profit at their expense. Our analysis indicates that a broker can also trade based on knowledge of the trades of his liquidity-motivated customers and earn positive profits. Such trading, however, benefits the liquidity traders because competition among dual

[8] The intuition behind our results differs from Roell (1990) and Admati and Pfleiderer (1991) because our comparison has held the variance of liquidity trade fixed. In their models, revelations concerning liquidity trade decrease the expected profits agents earn on private information concerning the security's fundamental value. This is because the dual trader's orders or sunshine trader's announcement decreases the marketmaker's beliefs about the variance of liquidity trade, which decreases the rents the informed earn on their private information about fundamentals. Consequently, the expected losses borne by liquidity traders decrease.

traders reduces the aggregate expected profits earned at the expense of liquidity traders.[9] If the marketmaker were to update his beliefs about liquidity trade from order flow, the benefits associated with dual trading to liquidity traders would be less than implied by our model. This is because (by Definition 1) when the marketmaker does not observe μ_x, the price he sets does not reflect the extent to which the net order flow he observes is related to liquidity trade. Dual trading decreases net order flow because dual traders submit orders that offset liquidity trades. This leads to more favorable prices for the execution of liquidity orders. Instead, if the marketmaker were to update his beliefs about liquidity trade based on his observation of order flow, dual trading would lead to a smaller reduction in order imbalances and would have a smaller impact on reducing liquidity-motivated agents' expected losses. In addition to expected profits, our results suggest that dual trading decreases agents' incentives to acquire private information about security fundamentals because such information degrades the profitability of trading on knowledge of liquidity trade.

In a related paper, Pichler (1990) examines a market in which a monopolist specialist precommits to a single price (as opposed to bid and ask prices) at which he satisfies all orders arriving from potentially informed risk-averse agents. Pichler shows that if the specialist knows the direction of liquidity trade in advance, he will adjust the price he quotes to profit from this knowledge. This feature of pricing in her model is similar to our Proposition 3.2, except that her specialist quotes the same price for all levels of net order flow. She also shows that since the specialist quotes a single price regardless of the sign of order flow, the specialist's profit-maximizing price is closer to his expected value of the security the greater is adverse selction (i.e., the informational advantage that agents have over the specialist). The remainder of Pichler's analysis focuses on how prices and traders' expected utility change as the direction of liquidity trade or the degree of adverse selection changes. She does not compare differences across equilibria associated with different allocations of information about liquidity trade across agents. This contrasts with our approach; the stochastic structure governing liquidity trade and private information is held fixed across the cases we compare, and the only difference across economies is the allocation of information about liquidity trade.

[9] Our result is different from the one obtained by Fishman and Longstaff (1992) who also find that dual trading is desirable to exogenous liquidity traders. In their model, the dual trader/broker trades to maximize profits, but then charges (competitive) commissions that return those profits to their clients. Since dual trading imposes disproportional costs on the informed and the smaller commissions are spread equally across clients, the uninformed benefit at the expense of the informed. In the absence of dual trading, no such transfer from the informed to the uninformed occurs in their model.

4. ANONYMITY REGARDING THE MAGNITUDE OF LIQUIDITY TRADE

In this section, we model less than full anonymity by providing market participants with some knowledge of the magnitude of liquidity trade (but not its direction); we assume that expectations of the *square* of liquidity trade are revised to $\sigma_x \in \{\sigma_H, \sigma_L\}$. First, we analyze the case where the marketmaker's beliefs are revised and then the case where no revision occurs. In both cases, the M profit-maximizing agents' expectations of $\tilde{x}^2$ are revised with probability q. We then compare behavior in the equilibria in which q takes the values of zero and one for each case. This is followed by a comparison of the two cases relating to whether or not the magnitude is revealed to the marketmaker. There we discuss the informational aspects of the equilibria.

4.1. *Marketmaker Does Not Observe σ_x*

We assume that the marketmaker does not know whether σ_x takes the value σ_H or σ_L. The N agents who observe y, and $(M - N)$ agents who do not observe y, observe σ_x with probability q. In this context, there exists an equilibrium with linear trading strategies and a linear pricing rule.

PROPOSITION 4.1. *There exists an equilibrium in which*

$$P(\omega; \Psi) = \theta_o + \lambda_o \omega, \qquad \Psi(\Phi_i^A; P) = \beta_{\sigma_x} + \alpha_{\sigma_x} y, \qquad \Psi(\Phi_i^B; P) = \delta_{\sigma_x}.$$

Agents' optimal expected profits are given by

$$E[\pi_i(\psi_i^A)|\Phi_i^A; N, q] = \frac{I_v \sigma_v}{(N + 1)^2 \lambda_o}$$

$$E[\pi_i(\psi_i^B)|\Phi_i^B; N, q] = 0,$$

where $I_v = \sigma_v/(\sigma_v + \sigma_\varepsilon)$.

When the marketmaker is unaware of the magnitude of liquidity trade, prices do not depend on it. In particular, the sensitivity of prices to order flow is independent of this magnitude. This and risk neutrality imply that agents' behavior and equilibrium expected profits do not depend on whether the magnitude of liquidity trade is revealed ($\beta_{\sigma_x} = \beta_o = \delta_{\sigma_x} = \delta_o = 0$).[10] Since the direction of liquidity trade is held fixed, the marketmaker's unconditional expected profits are zero (as would be true if the marketmaker were to update his beliefs about σ_x based on order flow). This is

[10] The restriction to linear strategies is important here. Nonlinear strategies could allow agents to profit from observing the magnitude when the marketmaker does not observe it.

because the marketmaker's ignorance of the intensity of liquidity trade does not cause him to set prices that are biased, on average, against the liquidity traders.

4.2. *Marketmaker Observes σ_x*

We now assume that the marketmaker knows whether σ_x takes the value σ_H or σ_L. The N agents who observe y, and $(M - N)$ agents who do not observe y, observe σ_x with probability q. In this context, there exists an equilibrium with linear trading strategies and a linear pricing rule.

PROPOSITION 4.2. *There exists an equilibrium in which*

$$P(\omega; \Psi) = \theta_{\sigma_x} + \lambda_{\sigma_x}\omega, \qquad \Psi(\Phi_i^A; P) = \beta_{\sigma_x} + \alpha_{\sigma_x} y, \qquad \Psi(\Phi_i^B; P) = \delta_{\sigma_x}.$$

Agents' optimal expected profits are given by

$$E[\pi_i(\psi_i^A)|\Phi_i^A; N, q] = \frac{I_v\sigma_v}{(N+1)^2}\left[q\left(\frac{1}{2\lambda_{\sigma_H}} + \frac{1}{2\lambda_{\sigma_L}}\right) + \frac{(1-q)}{\bar{\lambda}}\right]$$

$$E[\pi_i(\psi_i^B)|\Phi_i^B; N, q] = 0,$$

where $I_v = \sigma_v/(\sigma_v + \sigma_\varepsilon)$ *and* $\bar{\lambda} = \frac{1}{2}\lambda_{\sigma_H} + \frac{1}{2}\lambda_{\sigma_L}$.

Risk neutrality implies that agents' objective functions do not depend directly on the magnitude of liquidity trade. Consequently, trades based only upon knowledge of this magnitude are not profitable ($\delta_{\sigma_x} = \beta_{\sigma_x} = 0$). However, since the magnitude of liquidity trade is revealed to the marketmaker, he uses it to better estimate the value of the security given the order flow he observes. Therefore, if revealing this magnitude to agents affects their behavior, it must occur indirectly through the dependence of the marketmaker's pricing rule on the magnitude of liquidity trade.

In the event that liquidity trade is anonymous to agents, they have no way of knowing which pricing rule (σ_H or σ_L) is being employed, so they base their behavior on the expected pricing rule. Alternatively, if the magnitude of liquidity trade is revealed to agents, they can use it to forecast the price of which their orders will clear. This is beneficial to agents with private information about security fundamentals because they can refine their trading strategies to better profit from their information. For this reason, the component of agents' orders that depends on private information changes according to whether the magnitude of liquidity trade is revealed and whether the level revealed is σ_H or σ_L ($\alpha_o \neq \alpha_{\sigma_H} \neq \alpha_{\sigma_L}$). Likewise, the equilibrium expected profits agents earn on their private information depends on whether the magnitude of liquidity trade is revealed. For the $(M - N)$ agents without private information, equilibrium expected profits are zero.

4.3. *Informational Aspects of Equilibrium*

We now compare the two cases presented above. The trading strategies and equilibrium expected profits in Propositions 4.1 and 4.2 indicate that agents neither trade nor earn profits based upon knowledge of the magnitude of liquidity trade alone. The only impact of such knowledge on agents' behavior and profits occurs indirectly through the manner in which they trade on private information about the security's fundamental value. This suggests that the anonymity feature of a market's design has important implications for the value of private information and the manner in which private information affects trading. This is only important if the magnitude of liquidity trade affects prices, however. If this magnitude is not revealed to the marketmaker, then agents' trading strategies and equilibrium expected profits are independent of whether liquidity trade is revealed to them.

Since prices depend on the magnitude of liquidity trade when it is revealed to the marketmaker, each agent who also knows this magnitude is better able to forecast prices and to profit from his information about the security's value. The marketmaker responds by increasing the sensitivity of prices to order flow to maintain expected profits of zero.

PROPOSITION 4.3 *If the magnitude of liquidity trade is not revealed to the marketmaker* ($\sigma_x \notin \Phi_o$) *and/or to agents* ($q = 0$), *then prices are equally sensitive to order flow in equilibrium. However, if the magnitude of liquidity trade is revealed to the marketmaker* ($\sigma_x \in \Phi_o$) *and to agents* ($q = 1$), *then equilibrium prices are more sensitive to order flow on average. That is,*

$$\lambda_o(1|\sigma_x \notin \Phi_o) = \lambda_o(0|\sigma_x \notin \Phi_o) = \bar{\lambda}(0|\sigma_x \in \Phi_o) < \bar{\lambda}(1|\sigma_x \in \Phi_o),$$

where $\lambda_o(q|\cdot) = \lambda_o|_q$ *and* $\bar{\lambda}(q|\cdot) = \frac{1}{2}\lambda_{\sigma_H}(q|\cdot) + \frac{1}{2}\lambda_{\sigma_L}(q|\cdot)$.

The collective ability of agents to forecast prices and refine their trading strategies when the magnitude of liquidity trade is revealed intensifies competition for the profits associated with private information. This competition effect is absent when private information is held monopolistically, but dominates when the number of privately informed agents is large. Consequently, revealing the magnitude of liquidity trade is associated with small (large) expected profits for privately informed agents when N is large (small). This also means that revealing the magnitude of liquidity trade yields small (large) expected losses for liquidity-motivated agent when N is large (small).

To characterize the effect of this tradeoff on equilibrium expected profits, we note that profits are inversely related to the sensitivity of the security price to order flow in equilibrium—the λ parameter. When agents observe σ_x, their expected profits depend on the average of the inverses (of λs) relating to the environments in which they trade (i.e., $\sigma_x = \sigma_H$ and $\sigma_x = \sigma_L$). When they do not observe σ_x, however, agents' expected profits depend on the inverse of the average λ relating to the environments in which they trade [see the expressions for expected profits in Proposition 4.2]. Our next result says that the relevant inverse is less when agents observe σ_x then when they do not, provided that N is beyond a certain bound; the converse is also true.

LEMMA 4.4. *Suppose the magnitude of liquidity trade is revealed to the marketmaker (i.e., $\sigma_x \in \Phi_o$). There exists a finite $\overline{N} > 2$ such that*

$$\frac{1}{2\lambda_{\sigma_H}(1)} + \frac{1}{2\lambda_{\sigma_L}(1)} < \frac{1}{\bar{\lambda}(0)}$$

if and only if $N \geq \overline{N}$, where $\bar{\lambda}(0) = \frac{1}{2}\lambda_{\sigma_H}(0) + \frac{1}{2}\lambda_{\sigma_L}(0)$.

Using this Lemma and Proposition 4.2, it is an easy exercise to prove what we claimed about expected profits. This result augments our earlier discussion comparing expected profits across equilibria.

PROPOSITION 4.5. *Suppose the magnitude of liquidity trade is revealed to the marketmaker. If $N > \overline{N}$, then informed agents' expected profits (and liquidity-motivated agents' expected losses) in equilibrium are less when σ_x is revealed to informed agents than when it is not. However, if $N < \overline{N}$, then informed agents' expected profits (and liquidity-motivated agents' expected losses) in equilibrium are greater when σ_x is revealed to informed agents than when it is not.*

This result has several implications for institutional arrangements in financial markets. Our earlier results suggest that liquidity-motivated agents can benefit from sunshine trading by disclosing the direction of the trade they plan to make. In markets where competition among informed agents is intense, a further benefit can be derived from disclosing the size of the trade. Alternatively, if competition is weak, disclosing the size of the trade increases the expected cost of executing liquidity trades. A similar comparison is relevant to determining the desirability of an open limit order book. If competition among informed traders is intense, the expected cost of executing liquidity trades would be less if profit-maximizing agents were to observe the sizes of order imbalances at various prices in the limit order book. However, if competition is weak, this practice would increase transaction costs for liquidity traders. Assuming that low-cost execution of liquidity-motivated orders is a desirable element of a market's design, this result suggests that whether the limit order book should be open or closed depends on the composition of the trading crowd. During time periods (or for specific securities) in which monopolistic possession of private information is present, a closed limit order book provides lower-cost execution of liquidity-motivated orders.

Changes in the variance of liquidity trade in Kyle (1985) models are offset by a change in the intensity of trading by informed agents in a manner that keeps price informativeness unchanged. Therefore, in the market where the magnitude of liquidity trade is revealed to the marketmaker and to the agents, price informativeness does not depend on the value taken by σ_x. In our model, prices are no less informative when agents do not observe the value taken by σ_x. This is because agents base their trading decisions on the expected sensitivity of prices to order flow. Informed agents who do not observe the magnitude of liquidity trade submit orders that are either too aggressive or too passive, relative to the order they would submit if they knew the realized value of σ_x. When they trade too aggressively (when $\sigma_x = \sigma_L$), prices are more informative than they would be if agents knew that $\sigma_x = \sigma_L$; when they trade too passively (when $\sigma_x = \sigma_H$), prices are less informative than they would be if agents knew that $\sigma_x = \sigma_H$. On average, these offset each other.

PROPOSITION 4.6. *Price informativeness is independent of whether, and to whom, the magnitude of liquidity trade is revealed.*

The results above apply to the situation where the number of privately informed agents is exogenous. When the magnitude of liquidity trade is observed by the marketmaker, profit-maximizing agents' profits depend on whether they observe it as well. Consequently, their incentive to acquire private information is affected by whether the magnitude of liquidity trade is revealed. Our final result addresses this issue.

PROPOSITION 4.7. *Assume that the magnitude of liquidity trade is revealed to the marketmaker. If c is large then $N^*(1; c) > N^*(0; c)$; if c is small then $N^*(1; c) \leq N^*(0; c)$. Alternatively, if the magnitude of liquidity trade is not revealed to the marketmaker, the equilibrium in private information acquisition is independent of whether $q = 0$ or $q = 1$.*

This result contrasts sharply with the uniformly negative incentive effect on private information acquisition associated with revealing the direction of liquidity trade. When private information is very costly, it is held monopolistically. In this case, the provision of less anonymity regarding the magnitude of liquidity trades is useful in stimulating the acquisition of private information about security value. This is because revealing the magnitude does not sufficiently intensify competition among those who are privately informed. Conversely, if private information is inexpensive, many agents acquire it. In this case, less anonymity about liquidity trades discourage additional agents from acquiring private information that is already widely held.[11] Consequently, revealing information about the vol-

[11] If private information about security value had been modeled as signals with idiosyncratic noise, a larger value of N would be required to induce the same amount of competition among informed traders that is characterized in Proposition 4.4. In this setting, the range of

ume of trade at different prices implied by the limit order book might be an effective way to encourage the production of new information.

Our analysis focuses on comparing markets that have identical stochastic structures, but differ according to the allocation across traders of knowledge of the magnitude of liquidity trade. In a related paper, Lindsey (1990) compares markets that differ according to the probability distribution from which liquidity trade is drawn. Specifically, he compares a market (like ours) in which liquidity trade is drawn from a mixture of normal distributions that have variances of σ_H and σ_L to a market (unlike ours) in which liquidity trade is drawn from a normal distribution that has variance $\bar{\sigma} = \frac{1}{2}\sigma_H + \frac{1}{2}\sigma_L$. In the former case, the marketmaker is assumed to know whether liquidity trade is drawn from the σ_H or σ_L distribution, but other agents do not know. In the latter case, liquidity trade is drawn from the $\bar{\sigma}$ distribution for sure, so liquidity trade is not drawn from a mixture of normal distributions. Consequently, while Lindsey's markets do have different allocations of information about the variance of liquidity trade, they also have different assumptions about the stochastic structure of liquidity trade.

This aspect of modeling strategy has important implications for the results. Since $N = 1$ is assumed throughout Lindsey's (1990) model, the main difference between his and our results can be illustrated by comparing the inequality in Proposition 4.3 with a result reported by Lindsey (1990). He finds that $\bar{\lambda}(0|\sigma_x \in \Phi_o)$ is greater than the value of $\lambda(\bar{\sigma})$ that prevails in an economy where $\tilde{x} \sim N(0, \bar{\sigma})$ and suggests the interpretation that prices are less sensitive when the volatility of liquidity trade is common knowledge. The former statistic, $\bar{\lambda}(0|\sigma_x \in \Phi_o)$, is an *average computed from two possible prices*, whereas the latter statistic, $\lambda(\bar{\sigma})$, is not an average, but relates to a single price set by a marketmaker when the *variance of liquidity trade takes an average value*. Lindsey's result arises from Jensen's inequality and the fact that λ's are inversely related to σ_x—i.e., the average inverse σ_x is greater than the inverse of the average σ_x. By contrast, our result, which holds the stochastic structure of liquidity trade fixed, indicates that prices are more sensitive to order flow when the volatility of liquidity trade is common knowledge.

5. CONCLUSION

In this paper, we model risk-neutral securities markets that differ with regard to whether liquidity trades can be anonymously executed. Information about liquidity trade is modeled as a revelation that leads to a

---

values of c for which revealing the magnitude of liquidity trade discourages the acquisition of private information would be smaller.

change in the marketmaker's and/or agents' beliefs regarding liquidity trade. We show that the degree of anonymity provided by a market is important to market participants' expected profits and several aspects of market quality. Therefore, our results are relevant to policymakers who are concerned with agents' expected profits and market quality. In addition, our analysis has implications for several issues of market design including the automation of trading and the decentralization of markets.

The extent to which a market provides anonymous execution of liquidity-motivated orders affects the costs of order execution, the depth of the market, and the production of private information about a security's fundamental value. Our results suggest that different configurations of the anonymity feature of a market's design can be used to achieve different policy objectives. For example, revealing the magnitude of liquidity trade to all market participants encourages agents to acquire costly private information and discourages the acquisition of inexpensive information that is already widely distributed among market participants. Alternatively, maintaining anonymity with respect to the direction of liquidity trade increases agents' incentives to acquire private information, regardless of cost.

Our results have implications for the institutional arrangements of sunshine trading and dual trading. Order execution costs for liquidity traders is lower if they disclose the direction of their trades in advance. However, a disclosure of the size of their trade only decrease transactions costs if there is sufficient competition among agents with private information about the security's fundamental value. We also show that the practice of dual trading can decrease order execution costs faced by liquidity-motivated agents, but that dual trading decreases agents' incentives to acquire private information about fundamentals.

Automation of trading systems and decentralization of markets are among the most discussed current developments in domestic and international finance. Automated trading systems can simply improve the execution of orders in existing organized exchanges or abolish floor trading entirely in favor of screen-driven computerized trading systems. Specifying the information to be revealed on screen is crucial to the design of an automated system [see Domowitz (1990) for an analysis of three automated systems]. A potentially important part of that information is the identity of the agents participating in the market and the resulting order flow. The same issue is important in deciding whether trading of a security should be done on centralized or decentralized markets. Our results provide some clarification of these issues.

If by knowing who is present on the trading floor (or actively monitoring a computer screen) agents are better able to infer something about the magnitude of liquidity trades, then centralization (or disclosing the identity of current participants in an automated system) can provide lower execution costs for liquidity-motivated orders than a decentralized market, when competition among informed agents is intense. Conversely, in markets where private information is held monopolistically, a decentralized market structure that discloses little or nothing about agents' willingness and motivation to trade achieves lower costs of executing liquidity-motivated orders. If a firm's management chooses where to list its securities based on minimizing the cost of executing liquidity trades, then securities of firms where large numbers of investors are informed would be listed on centralized markets (like the NYSE/AMEX). Alternatively, securities of firms where few investors are informed would be listed on decentralized markets (like the OTC market). A similar trade-off is relevant for choosing among automated trading systems. Firms would choose to list their stocks on automated trading systems with differing degrees of disclosure according to whether private information is widely distributed among traders or monopolistically held.

APPENDIX

Proof of Proposition 3.1. If agent $i \in A$ observes μ_x then his optimal order, ψ_i^A, solves

$$\max_{\psi_i^A} E[\psi_i^A(\tilde{v} - \theta_{\mu_x} - \lambda_{\mu_x}\tilde{\omega}) | y, \mu_x],$$

where $\omega = \psi_i^A + \Sigma_{j\in A_{-i}}\psi_j^A + \Sigma_{k\in B}\psi_k^B + x$. So ψ_i^A maximizes

$$\psi_i^A I_v y - \theta_{\mu_x}\psi_i^A - \lambda_{\mu_x}(\psi_i^A)^2 - \lambda_{\mu_x}\psi_i^A \sum_{j\in A_{-i}} \psi_j^A - \lambda_{\mu_x}\psi_i^A \sum_{k\in B} \psi_k^B - \lambda_{\mu_x}\psi_i^A\mu_x,$$

where $I_v = \sigma_v/(\sigma_v + \sigma_\varepsilon)$. The first-order condition implies

$$\psi_i^A = \frac{1}{2\lambda_{\mu_x}}\left\{I_v - \lambda_{\mu_x}\sum_{j\in A_{-i}}\alpha_{j,\mu_x}\right\} y - \frac{1}{2\lambda_{\mu_x}}\left\{\theta_{\mu_x} + \lambda_{\mu_x}\left(\sum_{j\in A_{-i}}\beta_{j,\mu_x} + \sum_{k\in B}\delta_{k,\mu_x} + \mu_x\right)\right\},$$

where A_{-i} is the subset of A consisting of all elements except element i. The only solution to this system of equations (one for each agent i) is symmetric—i.e., $\alpha_{i,\mu_x} = \alpha_{\mu_x}$, $\beta_{i,\mu_x} = \beta_{\mu_x}$ for all i—implying that

$$\alpha_{\mu_x} = \frac{I_v}{(N+1)\lambda_{\mu_x}}$$

and

$$\beta_{\mu_x} = -\left(\frac{1}{N+1}\right)\left\{\frac{\theta_{\mu_x}}{\lambda_{\mu_x}} + \sum_{k\in B}\delta_{k,\mu_x} + \mu_x\right\}. \tag{3.1.1}$$

Similar reasoning implies that when μ_x is observed, the order of agent $i \in B$ is given by

$$\psi_i^B = -\frac{1}{2\lambda_{\mu_x}}\left\{\theta_{\mu_x} + \lambda_{\mu_x}\left(N\beta_{\mu_x} + \sum_{k\in B_{-i}}\delta_{k,\mu_x} + \mu_x\right)\right\}.$$

The only solution to this system is symmetric—i.e., $\delta_{i,\mu_x} = \delta_{\mu_x}$ for all i—implying that

$$\delta_{\mu_x} = -\left(\frac{1}{M-N+1}\right)\left\{\frac{\theta_{\mu_x}}{\lambda_{\mu_x}} + N\beta_{\mu_x} + \mu_x\right\}. \tag{3.1.2}$$

Combining (3.1.1) and (3.1.2) yields

$$\delta_{\mu_x} = \beta_{\mu_x} = -\frac{1}{M+1}\left(\frac{\theta_{\mu_x}}{\lambda_{\mu_x}} + \mu_x\right) \text{ for } \mu_x \in \{\mu_H, \mu_L\}.$$

This completes the analysis of agents' behavior when μ_x is observed.

If agent $i \in A$ does not observe μ_x then his order, ψ_i^A, solves

$$\max_{\psi_i^A} E[\psi_i^A(\tilde{v} - \theta_{\mu_x} - \lambda_{\mu_x}\tilde{\omega})|\,y],$$

where $\omega = \psi_i^A + \sum_{j\in A_{-i}}\psi_j^A + \sum_{k\in B}\psi_k^B + x$. So ψ_i^A maximizes

$$\psi_i^A I_v y - \bar{\theta}\psi_i^A - \bar{\lambda}(\psi_i^A)^2 - \bar{\lambda}\psi_i^A\sum_{j\in A_{-i}}\psi_j^A - \bar{\lambda}\psi_i^A\sum_{k\in B}\psi_k^B,$$

where $\bar{\lambda} = \frac{1}{2}\lambda_{\mu_H} + \frac{1}{2}\lambda_{\mu_L}$, and $\bar{\theta} = \frac{1}{2}\theta_{\mu_H} + \frac{1}{2}\theta_{\mu_L}$. The first-order condition implies

$$\psi_i^A = \frac{1}{2\bar{\lambda}}\left\{I_v - \bar{\lambda}\sum_{j\in A_{-i}}\alpha_{j,o}\right\}y - \frac{1}{2\bar{\lambda}}\left\{\bar{\theta} + \bar{\lambda}\left(\sum_{j\in A_{-i}}\beta_{j,o} + \sum_{k\in B}\delta_{k,o} + \mu_x\right)\right\}.$$

The only solution is symmetric—$\alpha_{i,\mu_x} = \alpha_{\mu_x}$ and $\beta_{i,\mu_x} = \beta_{\mu_x}$ for all i—so

$$\alpha_o = \frac{I_v}{(N+1)\bar{\lambda}}$$

and

$$\beta_o = -\left(\frac{1}{N+1}\right)\left\{\frac{\bar{\theta}}{\bar{\lambda}} + \sum_{k\in B} \delta_{k,o}\right\}. \tag{3.1.3}$$

Similar reasoning implies that when μ_x is not observed, the order of agent $i \in B$ is given by

$$\psi_i^B = -\frac{1}{2\bar{\lambda}}\left\{\bar{\theta} + \bar{\lambda}\left(N\beta_o + \sum_{k\in B_{-i}} \delta_{k,o}\right)\right\},$$

where B_{-i} is the subset of B consisting of all elements except element i. The only solution is symmetric—$\delta_{i,o} = \delta_o$ for all i—so

$$\delta_o = -\frac{1}{M-N+1}\left\{\frac{\bar{\theta}}{\bar{\lambda}} + N\beta_o\right\}. \tag{3.1.4}$$

Combining (3.1.3) and (3.1.4) yields

$$\beta_o = \delta_o = -\frac{1}{M+1}\left(\frac{\bar{\theta}}{\bar{\lambda}}\right).$$

Since β's and δ's are always equal, we henceforth use only β's.

Conditional on the value of μ_x, the marketmaker's inference problem involves normal variates $\{\tilde{v}, \tilde{\omega}\}$:

$$P(\omega|\mu_x) = \frac{\operatorname{Cov}[\tilde{v}, \tilde{\omega}|\mu_x]}{\operatorname{Var}[\tilde{\omega}|\mu_x]}(\omega - E[\tilde{\omega}|\mu_x]) \equiv \theta_{\mu_x} + \lambda_{\mu_x}\omega,$$

where, conditional on μ_x, $\tilde{\omega}$ is given by

$$\tilde{\omega} = \begin{cases} \tilde{\omega}_{\mu_x} \equiv M\beta_{\mu_x} + N\alpha_{\mu_x}\tilde{y} + \tilde{x} & \text{with probability } q \\ \tilde{\omega}_o \equiv M\beta_o + N\alpha_o\tilde{y} + \tilde{x} & \text{with probability } 1-q. \end{cases}$$

From this we compute the following moments:

$$\begin{aligned}
\operatorname{Cov}[\tilde{v}, \tilde{\omega}|\mu_x] &= N\sigma_v[q\alpha_{\mu_x} + (1-q)\alpha_o] \\
\operatorname{Var}[\tilde{\omega}|\mu_x] &= qE[\tilde{\omega}_{\mu_x}^2|\mu_x] + (1-q)E[\tilde{\omega}_o^2|\mu_x] - \{qE[\tilde{\omega}_{\mu_x}|\mu_x] \\
&\quad + (1-q)E[\tilde{\omega}_o|\mu_x]\}^2 \\
E[\tilde{\omega}_{\mu_x}^2|\mu_x] &= [M\beta_{\mu_x} + \mu_x]^2 + N^2\alpha_{\mu_x}^2(\sigma_v + \sigma_\varepsilon) + \sigma \\
E[\tilde{\omega}_o^2|\mu_x] &= [M\beta_o + \mu_x]^2 + N^2\alpha_o^2(\sigma_v + \sigma_\varepsilon) + \sigma \\
E[\tilde{\omega}_{\mu_x}|\mu_x] &= \mu_x + M\beta_{\mu_x} \\
E[\tilde{\omega}_o|\mu_x] &= \mu_x + M\beta_o.
\end{aligned}$$

Substituting and rearranging yields

$$\lambda_{\mu_x} = N\sigma_v\alpha/(q(M\beta_{\mu_x} + \mu_x)^2 + (1 - q)(M\beta_o + \mu_x)^2 + N^2\alpha^2(\sigma_v + \sigma_\varepsilon) + \sigma - [qM\beta_{\mu_x} + (1 - q)M\beta_o + \mu_x]^2)$$

for $\mu_x \in \{\mu_H, \mu_L\}$, having used the fact that $\alpha_{\mu_H} = \alpha_{\mu_L} = \alpha_o \equiv \alpha$. We also have $\theta_{\mu_x} = -\lambda_{\mu_x}\hat{\theta}_{\mu_x}$, where $\hat{\theta}_{\mu_x} \equiv E[\tilde{\omega}|\mu_x] = qE[\tilde{\omega}_{\mu_x}|\mu_x] + (1 - q)E[\tilde{\omega}_o|\mu_x]$ for $\mu_x \in \{\mu_H, \mu_L\}$; substitution and rearrangement yields

$$\hat{\theta}_{\mu_H} = \mu_H - \frac{N}{N+1} q(\mu_H - \hat{\theta}_{\mu_H}) - \frac{N(1-q)}{2(N+1)\bar{\lambda}} [\lambda_{\mu_H}(\mu_H - \hat{\theta}_{\mu_H}) + \lambda_{\mu_L}(\mu_L - \hat{\theta}_{\mu_L})]$$

$$\hat{\theta}_{\mu_L} = \mu_L - \frac{N}{N+1} q(\mu_L - \hat{\theta}_{\mu_L}) - \frac{N(1-q)}{2(N+1)\bar{\lambda}} [\lambda_{\mu_H}(\mu_H - \hat{\theta}_{\mu_H}) + \lambda_{\mu_L}(\mu_L - \hat{\theta}_{\mu_L})].$$

Direct solution of this linear system yields the unique solution: $\{\hat{\theta}_{\mu_H} = \mu_H, \hat{\theta}_{\mu_L} = \mu_L\}$. Therefore, upon substituting this into the equations for α, the β's, and the λ's, we have

$$\alpha_{\mu_H} = \alpha_{\mu_L} = \alpha_o = \frac{I_v}{\lambda(N+1)}$$

$$\beta_{\mu_H} = \beta_{\mu_L} = \beta_o = 0$$

$$\theta_{\mu_x} = -\lambda\mu_x$$

$$\lambda \equiv \lambda_{\mu_H} = \lambda_{\mu_L} = \frac{1}{N+1}\left(\frac{NI_v\sigma_v}{\sigma}\right)^{1/2}.$$

Finally, substituting the optimal ψ_i's into the expected profit equations and taking expectations yield

$$E[\pi_i(\psi_i^A)|\Phi_i^A; N, q] = \frac{I_v\sigma_v}{\lambda(N+1)^2}$$

$$E[\pi_i(\psi_i^B)|\Phi_i^B; N, q] = 0$$

Proof of Proposition 3.2. The solution for agents' behavior is identical to the argument made in the Proof of Proposition 3.1 except that in the present case, λ and θ are not μ_x-contingent. This yields the relations that hold in equilibrium,

$$\alpha_{\mu_H} = \alpha_{\mu_L} = \alpha_o = \frac{I_v}{(N+1)\lambda_o}$$

$$\beta_{\mu_x} = \delta_{\mu_x} = -\left(\frac{1}{M+1}\right)\left(\frac{\theta_o}{\lambda_o} + \mu_x\right)$$

$$\beta_o = \delta_o = -\frac{1}{M+1}\left(\frac{\theta_o}{\lambda_o}\right),$$

where $I_v = \sigma_v/(\sigma_v + \sigma_\varepsilon)$. Since β's and δ's are always equal, we henceforth use only β's.

Conditional on the value of μ_x, the marketmaker's inference problem involves normal variates $\{\tilde{v}, \tilde{\omega}\}$. Using Definition 1,

$$P(\omega) = \theta_o + \lambda_o\omega = \tfrac{1}{2}E[\tilde{v}|\omega, \mu_H] + \tfrac{1}{2}E[\tilde{v}|\omega, \mu_L],$$

where, conditional on μ_x, $\tilde{\omega}$ is given by

$$\tilde{\omega} = \begin{cases} \tilde{\omega}_{\mu_x} \equiv M\beta_{\mu_x} + N\alpha_{\mu_x}\tilde{y} + \tilde{x} & \text{with probability } q \\ \tilde{\omega}_o \equiv M\beta_o + N\alpha_o\tilde{y} + \tilde{x} & \text{with probability } 1-q, \end{cases}$$

and

$$E[\tilde{v}|\omega, \mu_x] = \frac{\operatorname{Cov}[\tilde{v}, \tilde{\omega}|\mu_x]}{\operatorname{Var}[\tilde{\omega}|\mu_x]}(\omega - E[\tilde{\omega}|\mu_x]) \equiv \hat{\lambda}_{\mu_x}(\omega - E[\tilde{\omega}|\mu_x]).$$

Thus,

$$\lambda_o = \tfrac{1}{2}\hat{\lambda}_{\mu_H} + \tfrac{1}{2}\hat{\lambda}_{\mu_L} \qquad \text{and} \qquad \theta_o = \tfrac{1}{2}(\hat{\lambda}_{\mu_H}E[\tilde{\omega}|\mu_H]) + \tfrac{1}{2}(\hat{\lambda}_{\mu_L}E[\tilde{\omega}|\mu_L]).$$

Computing these moments yields expressions identical to those computed in the Proof of Proposition 3.1. Using these expressions, we have $\lambda_o = \frac{1}{2}\hat{\lambda}_{\mu_H} + \frac{1}{2}\hat{\lambda}_{\mu_L}$, where

$$\hat{\lambda}_{\mu_x} = N\sigma_v\alpha/(q(M\beta_{\mu_x} + \mu_x)^2 + (1-q)(M\beta_o + \mu_x)^2 + N^2\alpha^2(\sigma_v + \sigma_\varepsilon) \\ + \sigma - [qM\beta_{\mu_x} + (1-q)M\beta_o + \mu_x]^2)$$

having used the fact that $\alpha_{\mu_H} = \alpha_{\mu_L} = \alpha_o \equiv \alpha$. Substituting for α's and β's, and simplifying considerably, one has

$$\hat{\lambda}_{\mu_x} = \frac{NI_v\sigma_v}{(N+1)\lambda_o\left[\mu_x^2 q(1-q)\left(\frac{M}{M+1}\right)^2 + N^2\frac{I_v\sigma_v}{(N+1)^2\lambda_o^2} + \sigma\right]}.$$

Recall that $\theta_o = \frac{1}{2}\hat{\lambda}_{\mu_H}E[\tilde{\omega}|\mu_H] + \frac{1}{2}\hat{\lambda}_{\mu_L}E[\tilde{\omega}|\mu_L]$. Substituting for the conditional expectations yields

$$\theta_o = -\tfrac{1}{2}\hat{\lambda}_{\mu_H}[\mu_H + M(q\beta_{\mu_H} + (1-q)\beta_o)] - \tfrac{1}{2}\hat{\lambda}_{\mu_L}[\mu_L + M(q\beta_{\mu_L} + (1-q)\beta_o)].$$

Substituting for the β's and rearranging yields

$$\theta_o = -[M(1-q) + 1](\tfrac{1}{2}\hat{\lambda}_{\mu_H}\mu_H + \tfrac{1}{2}\hat{\lambda}_{\mu_L}\mu_L).$$

Substituting the optimal ψ_i's into the expected profit equations and taking expectations yields

$$E[\pi_i(\psi_i^A)|\Phi_i^A; N, q] = \frac{I_v\sigma_v}{\lambda_o(N+1)^2} + \frac{1}{(M+1)^2}\left\{q\lambda_o\left[\frac{1}{2}\left(\frac{\theta_o}{\lambda_o} + \mu_H\right)^2 + \frac{1}{2}\left(\frac{\theta_o}{\lambda_o} + \mu_L\right)^2\right] + (1-q)\frac{\theta_o^2}{\lambda_o}\right\}$$

$$E[\pi_i(\psi_i^B)|\Phi_i^B; N, q] = \frac{1}{(M+1)^2}\left\{q\lambda_o\left[\frac{1}{2}\left(\frac{\theta_o}{\lambda_o} + \mu_H\right)^2 + \frac{1}{2}\left(\frac{\theta_o}{\lambda_o} + \mu_L\right)^2\right] + (1-q)\frac{\theta_o^2}{\lambda_o}\right\}.$$

Proof of Proposition 3.3. For jointly normal variates, the reduction in posterior variance of $\tilde{v}$ due to observing P is

$$\Delta_{\mu_x}(q; \Phi_o) \equiv \frac{\{\mathrm{Cov}[\tilde{P}, \tilde{v}]\}^2}{\mathrm{Var}[\tilde{P}]}$$

written as a function of q, the realization of μ_x, and the marketmaker's knowledge Φ_o.

If the marketmaker knows μ_x (i.e., $\mu_x \in \Phi_o$), then from Proposition 3.1 we have

$$\tilde{P} = \begin{cases} \theta_{\mu_H} + \lambda\tilde{\omega} & \text{if } \mu_x = \mu_H \\ \theta_{\mu_L} + \lambda\tilde{\omega} & \text{if } \mu_x = \mu_L, \end{cases}$$

where $\lambda \equiv \lambda_{\mu_H} = \lambda_{\mu_L}$ and

$$\tilde{\omega} = N\alpha\tilde{y} + \tilde{x}$$

for both $q = 0$ and $q = 1$, and $\alpha \equiv \alpha_o = \alpha_{\mu_H} = \alpha_{\mu_L}$. Computing moments yields

$$\begin{aligned} \mathrm{Cov}[\tilde{P}, \tilde{v}|\mu_H] &= \mathrm{Cov}[\tilde{P}, \tilde{v}|\mu_L] = \lambda N\alpha\sigma_v \\ \mathrm{Var}[\tilde{P}|\mu_H] &= \mathrm{Var}[\tilde{P}|\mu_L] = \lambda^2[N^2\alpha^2(\sigma_v + \sigma_\varepsilon) + \sigma] \end{aligned}$$

for both $q = 0$ and $q = 1$. Therefore,

$$\Delta_{\mu_H}(0|\mu_x \in \Phi_o) = \Delta_{\mu_L}(0|\mu_x \in \Phi_o) = \Delta_{\mu_H}(1|\mu_x \in \Phi_o) = \Delta_{\mu_L}(1|\mu_x \in \Phi_o). \tag{3.3.1}$$

If the marketmaker does not know μ_x (i.e., $\mu_x \notin \Phi_o$), then from Proposition 3.2 we have

$$\tilde{P} = \lambda_o\tilde{\omega},$$

where

$$\tilde{\omega} = \begin{cases} N\alpha\tilde{y} + \tilde{x} & \text{if } q = 0 \\ N\beta_{\mu_x} + N\alpha\tilde{y} + \tilde{x} & \text{if } q = 1, \end{cases}$$

and $\alpha \equiv \alpha_o = \alpha_{\mu_H} = \alpha_{\mu_L}$. The λ and α parameters of Proposition 3.2 are equal to those of Proposition 3.1. If $q = 0$, computing the moments yields

$$\begin{aligned} \mathrm{Cov}[\tilde{P}, \tilde{v}|\mu_H] &= \mathrm{Cov}[\tilde{P}, \tilde{v}|\mu_L] = \lambda N\alpha\sigma_v \\ \mathrm{Var}[\tilde{P}|\mu_H] &= \mathrm{Var}[\tilde{P}|\mu_L] = \lambda^2[N^2\alpha^2(\sigma_v + \sigma_\varepsilon) + \sigma]. \end{aligned}$$

If $q = 1$, computing the moments yields identical expressions. Thus,

$$\Delta_{\mu_H}(0|\mu_x \notin \Phi_o) = \Delta_{\mu_L}(0|\mu_x \notin \Phi_o) = \Delta_{\mu_H}(1|\mu_x \notin \Phi_o) = \Delta_{\mu_L}(1|\mu_x \notin \Phi_o). \tag{3.3.2}$$

Equations (3.3.1) and (3.3.2) imply that

$$\overline{\Delta}(0|\mu_x \in \Phi_o) = \overline{\Delta}(1|\mu_x \in \Phi_o) = \overline{\Delta}(0|\mu_x \notin \Phi_o) = \overline{\Delta}(1|\mu_x \notin \Phi_o),$$

where $\overline{\Delta}(q|\cdot) = \frac{1}{2}\Delta_{\mu_H}(q|\cdot) + \frac{1}{2}\Delta_{\mu_L}(q|\cdot)$.

Proof of Proposition 3.4. Define $\lambda(N) = \lambda_o|_{N,q=0} = \lambda_o|_{N,q=1}$ and, for simplicity, $N_o = N^*(0, c)$ and $N_1 = N^*(1, c)$. We prove the second part of the proposition first. From Proposition 3.2, we have the following expressions for optimal expected profits when the direction of liquidity trade is not revealed to the marketmaker:

$$E[\pi_i(\psi_i^A); N, 1] = \frac{I_v\sigma_v}{(N+1)^2\lambda(N)} + \frac{((1/2)\mu_H^2 + (1/2)\mu_L^2)\lambda(N)}{(M+1)^2}$$

$$E[\pi_i(\psi_i^B); N, 1] = \frac{((1/2)\mu_H^2 + (1/2)\mu_L^2)\lambda(N)}{(M+1)^2}$$

$$E[\pi_i(\psi_i^A); N, 0] = \frac{I_v\sigma_v}{(N+1)^2\lambda(N)}$$

$$E[\pi_i(\psi_i^B); N, 0] = 0.$$

Using these expressions, define the expected surplus to becoming informed

$$\Pi(N, q) = E[\pi_i(\psi_i^A); N, q] - c - E[\pi_i(\psi_i^B); N-1, q].$$

By the definition of N_o and N_1 as unique equilibria in information acquisition, we have

$$\Pi(N_o, 0) = \frac{I_v\sigma_v}{(N_o+1)^2\lambda(N_o)} - c \geq 0$$

$$\Pi(N_o+1, 0) = \frac{I_v\sigma_v}{(N_o+2)^2\lambda(N_o+1)} - c < 0$$

$$\Pi(N_1, 1) = \frac{I_v\sigma_v}{(N_1+1)^2\lambda(N_1)} + \frac{(1/2)\mu_H^2 + (1/2)\mu_L^2}{(M+1)^2}[\lambda(N_1) - \lambda(N_1 - 1)] - c \geq 0$$

$$\Pi(N_1+1, 1) = \frac{I_v\sigma_v}{(N_1+2)^2\lambda(N_1+1)} + \frac{(1/2)\mu_H^2 + (1/2)\mu_L^2}{(M+1)^2}[\lambda(N_1+1) - \lambda(N_1)] - c < 0.$$

First, suppose that $\lambda(N_1+1) - \lambda(N_1) \leq 0$; then $\Pi(N_o+1, 0) < 0$ implies that

$$\Pi(N_o+1, 1) < 0 \tag{3.4.1}$$

and either

$$\Pi(N_o, 1) \geq 0 \tag{3.4.2a}$$

or

$$\Pi(N_o, 1) < 0. \tag{3.4.2b}$$

If (3.4.2a), then (3.4.2a) and (3.4.1) imply that $N_o = N_1$. Alternatively, if (3.4.2b), then the fact that

$$\Pi(N_1, 1) \geq 0 \qquad \text{and} \qquad \Pi(N_1 + 1, 1) < 0$$

and uniqueness of N_1 imply that $N_o > N_1$.

Now suppose that $\lambda(N_1) - \lambda(N_1 - 1) \geq 0$; then $\Pi(N_o, 0) \geq 0$ implies that

$$\Pi(N_o, 1) \geq 0 \tag{3.4.3}$$

and either

$$\Pi(N_o + 1, 1) < 0 \tag{3.4.4a}$$

or

$$\Pi(N_o + 1, 1) \geq 0. \tag{3.4.4b}$$

If (3.4.4a), then (3.4.3) and (3.4.4a) imply that $N_o = N_1$. Alternatively, if (3.4.4b), then the fact that

$$\Pi(N_1, 1) \geq 0 \qquad \text{and} \qquad \Pi(N_1 + 1, 1) < 0,$$

and uniqueness of N_1, implies that $N_o + 1 < N_1 + 1$ or $N_o < N_1$.

The remaining logical possibility is that $\lambda(N_1) < \lambda(N_1 + 1)$ and $\lambda(N_1) < \lambda(N_1 - 1)$. Under this hypothesis, we have

$$\Pi(N_1, 0) > \Pi(N_1, 1) \geq 0, \tag{3.4.5}$$

where the first inequality follows from the fact that $\lambda(N_1) < \lambda(N_1 - 1)$ and the second follows from the definition of N_1. We also have

$$\Pi(N_1 + 1, 0) < \Pi(N_1 + 1, 1) < 0, \tag{3.4.6}$$

where the first inequality follows from the fact that $\lambda(N_1 + 1) > \lambda(N_1)$ and the second follows from the definition of N_1. Equations (3.4.5) and (3.4.6) together imply that N_1 satisfies the requirements for an equilibrium in information acquisition when $q = 0$; therefore $N_o = N_1$.

To prove the first part of the proposition, simply note that the expected surplus expressions are identical for $q = 0$ and $q = 1$ in the case where the marketmaker knows the direction of liquidity trade. Further, this expression is the same as the expression for the surplus above (marketmaker does not know μ_x) when $q = 0$.

LEMMA 3.5. *$\lambda_o'(N) \le 0$ for all $N \ge 1$, with equality if and only if $N = 1$; and $\lambda_o(1) > \lambda_o(0) = 0$.*

Proof of Lemma 3.5. From Proposition 3.2, for $q = 0$ and $q = 1$ equilibrium λ_o satisfies

$$2\sigma_x^2\lambda_o^2 + 2I_v\sigma_v\sigma_x\left(\frac{N(N-1)}{(N+1)^2}\right)\lambda_o - 2(I_v\sigma_v)^2\frac{N^3}{(N+1)^4} = 0.$$

Implicitly differentiating with respect to N and simplifying yields the condition

$$\lambda_o'(N) < 0 \qquad \text{if and only if } \left(\frac{I_v\sigma_v}{\sigma_x}\right)\frac{N^2(3-N)}{(N+1)^2(3N-1)} < 0.$$

This condition is clearly satisfied for $N \ge 3$. Direct substitution of $N = 1$ and $N = 2$ establishes that the condition is satisfied for $N = 2$ and holds with equality for $N = 1$.

Proof of Proposition 4.1. The solution for agents' behavior is identical to the argument made in the proof of Proposition 3.1 except that in the present case, $\mu_x = 0$, λ_o, and θ_o are not μ_x-contingent, and σ_x replaces σ. This yields the relations that hold in equilibrium,

$$\alpha_{\sigma_H} = \alpha_{\sigma_L} = \alpha_o = \frac{I_v}{(N+1)\lambda_o}$$

$$\beta_{\sigma_x} = \delta_{\sigma_x} = \beta_o = \delta_o = -\frac{1}{M+1}\left(\frac{\theta_o}{\lambda_o}\right),$$

where $I_v = \sigma_v/(\sigma_v + \sigma_\varepsilon)$. Since β's and δ's are always equal, we henceforth use only β's.

Conditional on the value of σ_x, the marketmaker's inference problem involves normal variates $\{\tilde{v}, \tilde{\omega}\}$. Using Definition 1,

$$P(\omega) = \theta + \lambda\omega = \tfrac{1}{2}E[\tilde{v}|\omega, \sigma_H] + \tfrac{1}{2}E[\tilde{v}|\omega, \sigma_L],$$

where

$$E[\tilde{v}|\omega, \sigma_x] = \frac{\mathrm{Cov}[\tilde{v}, \tilde{\omega}|\sigma_x]}{\mathrm{Var}[\tilde{\omega}|\sigma_x]}(\omega - E[\tilde{\omega}|\sigma_x]) \equiv \hat{\lambda}_{\sigma_x}(\omega - E[\tilde{\omega}|\sigma_x])$$

and where, conditional on σ_x, $\tilde{\omega}$ is given by

$$\tilde{\omega} = \begin{cases} \tilde{\omega}_{\sigma_x} \equiv M\beta_{\sigma_x} + N\alpha_{\sigma_x}\tilde{y} + \tilde{x} & \text{with probability } q \\ \tilde{\omega}_o \equiv M\beta_o + N\alpha_o\tilde{y} + \tilde{x} & \text{with probability } 1 - q. \end{cases}$$

Given this, we have

$$\lambda_o = \tfrac{1}{2}\hat{\lambda}_{\sigma_H} + \tfrac{1}{2}\hat{\lambda}_{\sigma_L}$$
$$\theta_o = -\tfrac{1}{2}\hat{\lambda}_{\sigma_H}E[\tilde{\omega}|\sigma_H] - \tfrac{1}{2}\hat{\lambda}_{\sigma_L}E[\tilde{\omega}|\sigma_L].$$

Computing these moments yields expressions identical to those computed in the Proof of Proposition 3.1, with $\mu_x = 0$, except that the conditioning set (and subscripts) are σ_x instead of μ_x. Using these expressions, we have $\lambda_o = \frac{1}{2}\hat{\lambda}_{\sigma_H} + \frac{1}{2}\hat{\lambda}_{\sigma_L}$, where, upon substituting, we find

$$\theta_o = 0 \qquad \text{and} \qquad \hat{\lambda}_{\sigma_x} = \frac{NI_v\sigma_v}{(N+1)\lambda_o[\sigma_x + N^2(I_v\sigma_v/(N+1)^2\lambda_o^2)]}.$$

Substituting the optimal choice into the expected profit equation yields optimal expected profits

$$E[\pi_i(\psi_i^A)|\Phi_i^A; N, q] = \frac{I_v\sigma_v}{(N+1)^2\lambda_o}$$
$$E[\pi_i(\psi_i^B)|\Phi_i^B; N, q] = 0.$$

Proof of Proposition 4.2. The solution for agents' behavior is identical to the argument made in the proof of Proposition 3.1 with $\mu_x = 0$ except that in the present case, the conditioning set (and subscripts) are σ_x instead of μ_x, λ is σ_x-contingent instead of μ_x-contingent, and σ_x replaces σ. This yields the relations that hold in equilibrium,

$$\alpha_{\sigma_x} = \frac{I_v}{(N+1)\lambda_{\sigma_x}}$$
$$\alpha_o = \frac{I_v}{(N+1)\bar{\lambda}}$$
$$\beta_{\sigma_x} = \delta_{\sigma_x} = -\frac{1}{M+1}\left(\frac{\theta_{\sigma_x}}{\lambda_{\sigma_x}}\right)$$
$$\beta_o = \delta_o = -\frac{1}{M+1}\left(\frac{\bar{\theta}}{\bar{\lambda}}\right),$$

where $\bar{\theta} = \frac{1}{2}\theta_{\sigma_H} + \frac{1}{2}\theta_{\sigma_L}$ and $\bar{\lambda} = \frac{1}{2}\lambda_{\sigma_H} + \frac{1}{2}\lambda_{\sigma_L}$. Since β's and δ's are always equal, we henceforth use only β's.

Conditional on the value of σ_x, the marketmaker's inference problem involves normal variates $\{\tilde{v}, \tilde{\omega}\}$:

$$P(\omega) = \frac{\mathrm{Cov}[\tilde{v}, \tilde{\omega}|\sigma_x]}{\mathrm{Var}[\tilde{\omega}|\sigma_x]}(\omega - E[\omega|\sigma_x]) \equiv \theta_{\sigma_x} + \lambda_{\sigma_x}\omega,$$

where, conditional on σ_x, $\tilde{\omega}$ is given by

$$\tilde{\omega} = \begin{cases} \tilde{\omega}_{\sigma_x} \equiv M\beta_{\sigma_x} + N\alpha_{\sigma_x}\tilde{y} + \tilde{x} & \text{with probability } q \\ \tilde{\omega}_o \equiv M\beta_o + N\alpha_o\tilde{y} + \tilde{x} & \text{with probability } 1 - q. \end{cases}$$

Computing these moments yields expressions identical to those computed in the Proof of Proposition 3.1, with $\mu_x = 0$. Using these expressions, we have $\theta_{\sigma_H} = \theta_{\sigma_L} = 0$. Solving for λ_{σ_x} yields

$$\lambda_{\sigma_x}^2 = \frac{NI_v\sigma_v\bar{\lambda}[q\bar{\lambda} + (1 - q)(N + 1)\lambda_{\sigma_x}]}{N^2I_v\sigma_v(1 - q) + \sigma_x(N + 1)^2\bar{\lambda}^2}.$$

Substituting the optimal choice into the expected profit equation yields optimal expected profits

$$E[\pi_i(\psi_i^A)|\Phi_i^A; N, q] = \frac{I_v\sigma_v}{(N + 1)^2}\left[q\left(\frac{1}{2\lambda_{\sigma_H}} + \frac{1}{2\lambda_{\sigma_L}}\right) + \frac{(1 - q)}{\bar{\lambda}}\right]$$

$$E[\pi_i(\psi_i^B)|\Phi_i^B; N, q] = 0.$$

Proof of Proposition 4.3. If $\sigma_x \notin \Phi_o$, then from Proposition 4.1, it is clear that λ_o is independent of q. Assume, instead, that $\sigma_x \in \Phi_o$. Since λ's are positive, $\bar{\lambda}(0) < \bar{\lambda}(1)$ if and only if $\bar{\lambda}(0)^2 < \bar{\lambda}(1)^2$. Upon substituting for these, where $\bar{\sigma} = \frac{1}{2}\sigma_H + \frac{1}{2}\sigma_L$, we have the equivalent expressions

$$\frac{1}{2\sigma_H\sigma_L}\{[\bar{\sigma}^2(N - 1)^2 + 4\sigma_H\sigma_L N]^{1/2} - \bar{\sigma}(N - 1)\}$$
$$< \frac{1}{4}\left[\frac{1}{\sigma_H} + \frac{1}{\sigma_L} + \frac{2}{(\sigma_H\sigma_L)^{1/2}}\right]$$

or

$$[\bar{\sigma}^2(N - 1)^2 + 4N\sigma_H\sigma_L] < \left[\frac{1}{2}(\sigma_H^{1/2} + \sigma_L^{1/2})^2 + \bar{\sigma}(N - 1)\right]^2$$

or

$$4N\sigma_H\sigma_L < \frac{1}{4}(\sigma_H^{1/2} + \sigma_L^{1/2})^4 + (\sigma_H^{1/2} + \sigma_L^{1/2})^2\bar{\sigma}(N-1)$$

and since $\sigma_L = \kappa\sigma_H$ for some $\kappa \in (0, 1)$,

$$16N\kappa < (1 + \kappa^{1/2})^4 + 2(N-1)(1 + \kappa^{1/2})^2(1 + \kappa)$$

or, after rearranging,

$$2N\{-\kappa^2 - 2\kappa^{3/2} + 6\kappa - 2\kappa^{1/2} - 1\} < -(1 + \kappa^{1/2})^2(1 - \kappa^{1/2})^2. \quad (4.3.1)$$

Note, however, that $(1 - \kappa^{1/2})^2 > 0$ holds for all $\kappa \in (0, 1)$. Squaring and multiplying by $2\kappa^{1/2}$ yields

$$2\kappa^{1/2} + 2\kappa^{3/2} - 4\kappa > 0,$$

therefore,

$$(1-\kappa)^2 + 2\kappa^{1/2} + 2\kappa^{3/2} - 4\kappa > (1-\kappa)^2$$

or, equivalently,

$$\{-\kappa^2 - 2\kappa^{3/2} + 6\kappa - 2\kappa^{1/2} - 1\} < -(1 + \kappa^{1/2})^2(1 - \kappa^{1/2})^2, \quad (4.3.2)$$

which holds for all $\kappa \in (0, 1)$. Since the value in $\{\cdot\}$ is negative by (4.3.2), (4.3.2) implies that (4.3.1) holds for all $N \geq 1$.

Proof of Lemma 4.4. Suppose $\sigma_x \in \Phi_o$; then from Proposition 4.2, for $q = 0$, the marketmaker's choice is

$$\lambda_{\sigma_x}(0) = \frac{N(N+1)I_v\sigma_v\bar{\lambda}(0)}{N^2I_v\sigma_v + \sigma_x(N+1)^2\bar{\lambda}(0)^2},$$

where $\bar{\lambda}(0) = \frac{1}{2}\lambda_{\sigma_H}(0) + \frac{1}{2}\lambda_{\sigma_L}(0)$. Rearranging and taking expectations yields the quadratic equation

$$\sigma_H\sigma_L(N+1)^4\bar{\lambda}(0)^4 + N(N+1)^2(N-1)I_v\sigma_v\bar{\sigma}\bar{\lambda}(0)^2 - N^3I_v^2\sigma_v^2 = 0,$$

where $\bar{\sigma} = \frac{1}{2}\sigma_H + \frac{1}{2}\sigma_L$. The unique positive solution to this is

$$\bar{\lambda}(0)^2 = \frac{NI_v\sigma_v}{2\sigma_H\sigma_L(N+1)^2}\{[(N_1)^2\bar{\sigma}^2 + 4N\sigma_H\sigma_L]^{1/2} - (N-1)\bar{\sigma}\}.$$

We also have

$$\lambda_{\sigma_H}(1)^2 = \frac{NI_v\sigma_v}{(N+1)^2\sigma_H} \quad \text{and} \quad \lambda_{\sigma_L}(1)^2 = \frac{NI_v\sigma_v}{(N+1)^2\sigma_L}.$$

The following expressions are equivalent:

$$\frac{1}{2\lambda_{\sigma_H}(1)} + \frac{1}{2\lambda_{\sigma_L}(1)} < \frac{1}{\bar{\lambda}(0)} \tag{4.4.1}$$

squaring

$$\bar{\lambda}(0)^2[\lambda_{\sigma_L}(1)^2 + 2\lambda_{\sigma_L}(1)\lambda_{\sigma_H}(1) + \lambda^2_{\sigma_H}(1)] < 4\lambda_{\sigma_L}(1)^2\lambda_{\sigma_H}(1)^2$$

substituting for the λ's

$$\frac{1}{2}\{[(N-1)^2\bar{\sigma}^2 + 4N\sigma_H\sigma_L]^{1/2} - (N-1)\bar{\sigma}\}\frac{(NI_v\sigma_v)^2}{\sigma_H\sigma_L(N+1)^4}$$
$$\left[\frac{1}{\sigma_H} + \frac{1}{\sigma_L} + \frac{2}{(\sigma_H\sigma_L)^{1/2}}\right] < \frac{4(NI_v\sigma_v)^2}{(N+1)^4\sigma_H\sigma_L}$$

or

$$[(N-1)^2\bar{\sigma}^2 + 4\sigma_H\sigma_L]^{1/2} - (N-1)\bar{\sigma} < 8\left\{\frac{\sigma_H\sigma_L}{\sigma_H + 2(\sigma_H\sigma_L)^{1/2} + \sigma_L}\right\}$$

squaring and rearranging

$$N(\sigma_H^{1/2} + \sigma_L^{1/2})^4 < 16\sigma_H\sigma_L + 4(N-1)\bar{\sigma}(\sigma_H^{1/2} + \sigma_L^{1/2})^2$$

considerable rearrangement yields

$$N > -\frac{16\sigma_H\sigma_L - 4\bar{\sigma}(\sigma_H^{1/2} - \sigma_L^{1/2})^2}{(\sigma_H^{1/2} + \sigma_L^{1/2})^2(\sigma_H^{1/2} - \sigma_L^{1/2})^2}$$

and since $\sigma_L = \kappa\sigma_H$ for some $\kappa \in (0, 1)$,

$$N > -\frac{16\kappa - 2(1+\kappa)(1+\kappa^{1/2})^2}{(1-\kappa)^2} \equiv \bar{N}(\kappa).$$

The right-hand side of this inequality can be shown to vary between the values of 2 and 3 as κ varies between 0 and 1. Since (4.4.1) is true if and only if this inequality is satisfied, we conclude that there is a $\overline{N} = \overline{N}(\kappa) > 2$ such that (4.4.1) is true if and only if $N \geq \overline{N}$.

Proof of Proposition 4.6. Define the equilibrium price of Proposition 4.2 as

$$P\sigma_x(q) = \lambda_{\sigma_x}(q)\omega_{\sigma_x}(q),$$

where the equilibrium order flow is

$$\omega_{\sigma_x}(q) = M\beta_{\sigma_x}(q) + N\alpha_{\sigma_x}(q) + x.$$

From Proposition 4.2 we can compute the parameter values

$$\alpha_{\sigma_x}(1) = \frac{I_v}{(N+1)\lambda_{\sigma_x}(1)}, \qquad \alpha_{\sigma_x}(0) = \frac{I_v}{(N+1)\bar{\lambda}(0)},$$
$$\lambda_{\sigma_x}(1)^2 = \frac{NI_v\sigma_v}{(N+1)^2\sigma_x},$$

where $\bar{\lambda}(0) = \frac{1}{2}\lambda_{\sigma_H}(0) + \frac{1}{2}\lambda_{\sigma_L}(0)$ computed from the expressions for $\lambda_{\sigma_x}(0)$ in Proposition 4.2. After simplification, one obtains

$$\bar{\lambda}(0)^2 = \frac{NI_v\sigma_v}{2\sigma_H\sigma_L(N+1)^2}\Sigma, \qquad \text{where}$$
$$\Sigma = \{[(N-1)^2\bar{\sigma}^2 + 4N\sigma_H\sigma_L]^{1/2} - (N-1)\bar{\sigma}\}, \tag{4.6.1}$$

where $\bar{\sigma} = \frac{1}{2}\sigma_H + \frac{1}{2}\sigma_L$. For jointly normal variates, the reduction in posterior variance of $\tilde{v}$ due to observing $P_{\sigma_x}(q)$ is

$$\Delta_{\sigma_x}(q) \equiv \frac{\{\mathrm{Cov}[\tilde{v}, \tilde{P}_{\sigma_x}(q)]\}^2}{\mathrm{Var}[\tilde{P}_{\sigma_x}(q)]}.$$

Using this definition and the equilibrium parameter values above, and simplifying, yields

$$\Delta(1) \equiv \Delta_{\sigma_x}(1) = \left(\frac{N}{N+1}\right) I_v\sigma_v$$

$$\Delta_{\sigma_x}(0) = \Delta(1)\left\{\frac{1}{\dfrac{N}{N+1} + \left(\dfrac{N+1}{N}\right)\dfrac{\bar{\lambda}(0)^2}{I_v\sigma_v}\sigma_x}\right\}.$$

From this it is clear that $\bar{\Delta}(0) = \Delta(1)$ if and only if

$$\frac{1}{2}\left\{\frac{1}{\frac{N}{N+1} + \left(\frac{N+1}{N}\right)\frac{\bar{\lambda}(0)^2}{I_v\sigma_v}\sigma_H}\right\} + \frac{1}{2}\left\{\frac{1}{\frac{N}{N+1} + \left(\frac{N+1}{N}\right)\frac{\bar{\lambda}(0)^2}{I_v\sigma_v}\sigma_L}\right\} = 1. \tag{4.6.2}$$

Considerable rearrangement yields the equivalent expression

$$\left(\frac{1-N}{N}\right)\frac{\bar{\lambda}(0)^2\bar{\sigma}}{I_v\sigma_v} = \left(\frac{N+1}{N}\right)^2\frac{\lambda(0)^4}{(I_v\sigma_v)^2}\sigma_H\sigma_L - \frac{N}{(N+1)^2}.$$

Substituting for $\bar{\lambda}(0)^2$ from (4.6.1) and rearranging yields the equivalent expression

$$2(1-N)\bar{\sigma}\Sigma = \Sigma^2 - 4N\sigma_H\sigma_L$$

implying that (4.6.2), and hence $\bar{\Delta}(0) = \Delta(1)$, are equivalent to

$$\Sigma^2 = 4N\sigma_H\sigma_L - 2(N-1)\bar{\sigma}\Sigma. \tag{4.6.3}$$

To verify that (4.6.3) holds, use the definition of Σ to compute Σ^2, which yields

$$\begin{aligned}\Sigma^2 &= \{2(N-1)\bar{\sigma}((N-1)\bar{\sigma} - [(N-1)^2\bar{\sigma}^2 + 4N\sigma_H\sigma_L]^{1/2}) + 4N\sigma_H\sigma_L\}\\ &= 4N\sigma_H\sigma_L - 2(N-1)\bar{\sigma}\Sigma,\end{aligned}$$

verifying (4.6.3) as desired.

Proof of Proposition 4.7. Assume that the magnitude of liquidity trade is revealed to the marketmaker. It is clear that for any q, $N^*(q; c)$ is decreasing in c. Therefore, we know that we can choose a c_1 sufficiently large to ensure that $N^*(0; c_1) \leq \bar{N}$. However, since $N^*(0; c_1) \leq \bar{N}$, we can show using Lemma 4.4. that

$$E[\pi_i(\psi_i^A)|\Phi_i^A; N^*(0; c_1), 1] > E[\pi_i(\psi_i^A)|\Phi_i^A; N^*(0; c_1), 0].$$

Since $E[\pi(\cdot|\cdot; N, q)]$ is decreasing in N for each q, we conclude that $N^*(1; c_1) > N^*(0; c_1)$. Using a similar argument, we can choose a c_2 sufficiently small to ensure that $N^*(1; c_2) > \bar{N}$, which in turn implies that $N^*(0; c_2) > N^*(1; c_2)$.

If the magnitude of liquidity trade is not revealed to the marketmaker, the private information component of expected profits is the same when $q = 0$ as it is when $q = 1$. Therefore, N^* is the same for these two cases.

REFERENCES

ADMATI, A., AND PFLEIDERER, P. (1988). A theory of intra-day patterns: Volume and price variability, *Rev. Finan. Stud.* **1,** 3–40.

ADMATI, A., AND PFLEIDERER, P. (1989). Divide and conquer: A theory of intraday and day-of-the-week mean effects, *Rev. Finan. Stud.* **2,** 189–223.

ADMATI, A., AND PFLEIDERER, P. (1991). Sunshine trading and financial market equilibrium, *Rev. Finan. Stud.* **4,** 443–481.

BLACK, F. (1992). "Equilibrium Exchanges," Working paper. Goldman Sachs Asset Management, New York, NY.

DOMOWITZ, I. (1990). The mechanics of automated trade execution systems, *J. Finan. Intermediation* **1,** 167–194.

EASLEY, D., AND O'HARA, M. (1987). Price, trade, size and information in securities markets, *J. Finan. Econ.* **19,** 69–90.

FISHMAN, M., AND HAGERTY, K. (1991). "The Mandatory Disclosure of Trades and Market Liquidity," Working paper. Northwestern University, Evanston, IL.

FISHMAN, M., AND LONGSTAFF, F. Dual trading in futures markets, *J. Finance* **47,** 643–671.

FOSTER, D., AND VISWANATHAN, S. (1990). A theory of the interday variations in volumes, variances and trading costs in securities markets, *Rev. Finan. Stud.* **3,** 593–624.

FOSTER, D., AND VISWANATHAN, S. (1991). "Market Microstructure Models with Elliptically Contoured Distributions," Working paper. Duke University, Durham, NC.

GLOSTEN, L. (1989). Insider trading, liquidity and the role of the monopolist specialist, *J. Bus.* **62,** 211–235.

GLOSTEN, L., AND MILGROM, P. (1985). Bid, ask and transactions prices in a specialist market with heterogeneously informed traders, *J. Finan. Econ.* **14,** 71–100.

HUGHSON, E. (1988). "Intraday Trade in Dealership Markets," Working paper. Carnegie-Mellon University, Pittsburgh, PA.

KUMAR, P., AND SEPPI, D. (1991). "Futures Manipulation with Cash Settlement," Working paper. Carnegie-Mellon University, Pittsburgh, PA.

KYLE, A. (1985). Continuous auctions and insider trading, *Econometrica* **53,** 1315–1335.

LINDSEY, R. (1990). "Market Markers, Asymmetric Information and Price Formation," Working paper. University of California, Berkeley, CA.

MADHAVAN, A. (1990). "Security Prices and Market Transparency," Working paper. University of Pennsylvania, Philadelphia, PA.

MADHAVAN, A., AND SMIDT, S. (1992). "An Analysis of Daily Changes in Specialist Inventories and Quotations," Working paper. University of Pennsylvania, Philadelphia, PA.

PICHLER, P. (1990). "Optimal Prices in Dealership Markets with Asymmetric Information," Working paper. University of Maryland, College Park, MD.

ROELL, A. (1990). Dual-capacity trading and the quality of the market, *J. Finan. Intermediation* **1,** 105–124.

SUBRAHMANYAM, A. (1991). A theory of trading in stock index futures, *Rev. Finan. Stud.* **4,** 17–51.